

Publications of Weinan E

Books

W. E, *Principles of Multiscale Modeling*, Cambridge University Press, 2011.

Papers

L. Wu, C. Ma and W. E, “How SGD Selects the Global Minima in Over-parameterized Learning: A Stability Perspective”, *NIPS*, 2018.

L. Zhang, J. Han, H. Wang, W. Saidi, R. Car and W. E, “End-to-end Symmetry Preserving Inter-atomic Potential Energy Model for Finite and Extended Systems”, *NIPS*, 2018.

Q. Li, L. Chen, C. Tai and W. E, “Maximum Principle Based Algorithms for Deep Learning”, *JMLR*, vol. 18, no. 165, pp. 1-29, 2018.

C. Ma, J.C. Wang and W. E, “Model reduction with memory and machine learning of dynamical systems”, *Comm. Comput. Phys.*, vol. 25, no. 4, pp. 947-962, 2019.

W. E, J. Han and Q. Li, “A Mean-Field Optimal Control Formulation of Deep Learning”, *Research in Mathematical Sciences*, vol. 6, no. 10, 2018.

L. F. Zhang, J. Han, R. Car, H. Wang and W. E, “Deep Potential Molecular Dynamics: A scalable model with the accuracy of quantum mechanics”, *Phys. Rev. Lett.*, vol. 120, no. 14, pp. 143001, 2018.

J. Han, A. Jentzen and W. E, “Solving high-dimensional partial differential equations using deep learning”, *Proc. Natl. Acad. Sci.*, vol. 115, no. 34, pp. 8505-8510, 2018.

J. Han, L. F. Zhang, R. Car and W. E, “Deep Potential: A General Representation of a Many-Body Potential Energy Surface”, *Comm. Comput. Phys.*, vol. 23, no. 3, pp. 629-639, 2018.

L. F. Zhang, H. Wang and W. E, “Reinforced dynamics for the enhanced sampling in large atomic and molecular systems. I. Basic Methodology”, *J. Chem. Phys.*, vol. 148, pp. 124113, 2018.

H. Wang, L. F. Zhang, J. Han and W. E, “DeePMD-kit: A deep learning package for many-body potential energy representation and molecular dynamics”, *Comput. Phys. Comm.*, vol. 228, pp. 178-184, 2018.

W. E, “A Proposal on Machine Learning via Dynamical Systems”, *Commun. Math. Stat.*, vol. 5, no. 1, pp.1-11, 2017.

W. E and B. Yu, “The Deep Ritz method: A deep learning-based numerical algorithm for solving variational problems”, *Comm. Math. Stats.*, vol. 6, no. 1, pp. 1-12, 2018.

W. E, J. Han and A. Jentzen, “Deep learning-based numerical methods for high-dimensional parabolic partial differential equations and backward stochastic differential equations”, *Comm. Math. Stats.*, vol. 5, no. 4, pp. 349-380, 2017.

- Q. Li, C. Tai and W. E, “Stochastic modified equations and the dynamics of stochastic gradient algorithms”, *JMLR*, vol. 20, no. 40, pp. 1-47, 2019.
- Q. Li, C. Tai and W. E, “Stochastic modified equations and adaptive stochastic gradient algorithms”, *International Conference of Machine Learning (ICML)*, 2017.
- W. E and Y. Wang, “Optimal convergence rates of the universal approximation error”, *Research in Mathematical Sciences*, vol. 4, no. 2, 2017.
- J. Han and W. E, “Deep learning approximation for stochastic control problems”, accepted, *NIPS Workshop on Deep Reinforcement Learning*, 2016.
- C. Tai and W. E, “Multi-scale adaptive representation of signals, I”, *JMLR*, vol. 17, no. 140, pp. 1-38, 2016.
- Q. Li, C. Tai and W. E, “Dynamics of stochastic gradient algorithms”, accepted by *International Conference of Machine Learning*.
- C. Tai, T. Xiao, X. Wang and W. E, “Convolutional neural networks with low-rank regularization”, arXiv:1511.06067, *ICLR*, 2016.
- W. E and Jianchun Wang, “A thermodynamic study of the two-dimensional pressure-driven channel flow”, *Discrete and Continuous Dynamical Systems*, vol. 36, no. 8, pp. 4339-4366, 2016.
- Jianchun Wang, Qianxiao Li and Weinan E, “Study of the instability of the Poiseuille flow using a thermodynamic formalism”, *Proceedings of the National Academy of Sciences*, 112(31), 2015.
- Q. Li and W. E, “The free action for non-equilibrium systems”, *J. Stat. Phys.*, vol. 161, no. 2, pp. 300-325, 2015.
- A. Samanta, M. Tuckerman, T. Yu and W. E, “Microscopic mechanisms of equilibrium melting of a solid,” *Science*, vol. 346, no. 6210, pp. 729-732, 2014.
- A. Samanta, W. E, M. Chen, T. Yu and M. Tuckerman, “Sampling the saddle points on free energy landscapes,” *J. Chem. Phys.*, vol. 140, no. 16, 164109, 2014.
- L. Lin, S. Shao and W. E, “Efficient iterative method for solving the Dirac-Kohn-Sham density functional theory,” *J. Comput. Phys.*, vol. 245, pp. 205-217, 2013.
- W. E and J. Lu, “The Kohn-Sham equation for deformed crystals,” *Memoire of the American Math Society*, 2012.
- L. Lin, J. Lu, L. Ying and W. E, “Optimized local basis set for Kohn-Sham density functional theory,” *J. Comput. Phys.*, vol. 231, no.13, pp. 4515-4529, 2012.
- W. E, X. Zhou and X. Cheng, “Subcritical bifurcations in spatially extended systems,” *Nonlinearity*, vol. 25, no.3, pp. 761-779, 2012.
- L. Lin, J. Lu, L. Ying and W. E, “Adaptive local basis set for Kohn-Sham density functional theory in a discontinuous Galerkin framework I: Total energy calculation,” *J. Comput. Phys.*, vol. 231, no. 4, pp. 2140-2154, 2012.

- A. Abdulle, W. E, B. Engquist and E. Vanden-Eijnden, “The heterogenous multiscale methods”, *Acta Numerica*, vol. 21, pp. 1-87, 2012.
- L. Lin, C. Yang, J. Lu, L. Ying and W. E, “A fast parallel algorithm for selected inversion of structured sparse matrix with application to 2D electronic structure calculations,” *SIAM J. Sci. Computing*, vol. 33, no.3, pp. 1329-1351, 2011.
- W. E and J. Lu, “The electronic structure of smoothly deformed crystals: Wannier functions and the Cauchy-Born rule,” *Arch. Ration. Mech. Anal.*, vol. 199, pp. 407–433, 2011.
- W. E and X. Zhou, “The gentlest ascent dynamics,” *Nonlinearity*, vol. 24, no. 6, pp. 1831-1842, 2011.
- W. E, J. Lu and X. Yang, “Effective Maxwell equations from time-dependent density functional theory”, *Acta Math. Sinica*, vol. 27, pp. 339–368, 2011.
- W. E, J. Lu and X. Yang, “Asymptotic analysis of the quantum dynamics: The Bloch-Wigner transform and Bloch dynamics”, *Acta. Appl. Math. Sinica.*, (25 July 2011), pp. 1–12.
- L. Lin, C. Yang, J. C. Meza, J.Lu, L. Ying and W. E, “SelInv – An algorithms for selected inversion of a sparse symmetric matrix”, *ACM Transactions on Mathematical Software*, vol. 37, no. 4, pp. 40:1-40:19, 2011.
- W. E, T. Li and J. Lu, “Localized basis of eigen-subspaces”, *Proc. Natl. Acad. Sci. USA*, vol. 109, pp. 1273–1278, 2010.
- W. E and E. Vanden-Eijnden, “The transition path theory and path-finding algorithms for the study of rare events,” *Ann. Rev. Phys. Chem*, vol. 61, pp. 391–420, 2010.
- W. E and J. Lu, “The electronic structure of smoothly deformed crystals: Cauchy-Born rule for the nonlinear tight-binding model,” *Comm. Pure Appl. Math.*, vol. 63, pp. 1432–1468, 2010.
- W. Ren, D. Hu and W. E, “Continuum theories for the moving contact line problem,” *Phys. Fluids*, vol. 22, pp. 102103, 2010.
- X. Cheng, L. Lin, W. E, A-C. Shi, and P. Zhang, “Nucleation of Ordered Phases in Block Copolymers,” *Phys. Rev. Lett.*, vol. 104, pp. 148301–148301-4, 2010.
- X. Li, J. Z. Yang, and W. E., ”A multiscale coupling method for the modeling of dynamics of solids with application to brittle cracks,” *J. Comput. Phys.*, vol. 229, no. 10, pp. 3970–3987, 2010.
- L. Lin, X. Cheng, W. E, A-C. Shi, and P. Zhang, ”A numerical method for the study of nucleation of ordered phases,” *J. Comput. Phys.*, vol. 229, no. 5, pp. 1797–1809, 2010.
- X. Wan, X. Zhou and W. E, “Study of noise-induced transitions in the Kuramoto-Sivashinsky equation via the minimum action method”, *Nonlinearity*, vol. 23, no. 3, pp. 475–494, 2010.
- X. Zhou and W. E, “Study of noise-induced transitions in the Lorenz system using the minimum action method,” *Comm. Math. Sci.*, vol. 8, pp. 341–355, 2010.
- L. Lin, C. Yang, J. Lu, L. Ying and W. E, “A fast parallel algorithm for selected inversion of structured sparse matrices with application to 2D electronic structure calculation,” *Lawrence Berkeley National Laboratory*. LBNL Paper LBNL-2677E. Retrieved from: <http://escholarship.org/uc/item/46q6w084>, 2010.

- L. Lin, J. Lu, L. Ying and W. E, “Pole-based approximation of the Fermi-Dirac function,” *Chin. Ann. Math.*, vol. 30B, pp. 729–742, 2009.
- W. E, W. Ren, and E. Vanden-Eijnden, “A general strategy for designing seamless multiscale methods,” *J. Comput. Phys.*, vol. 228, no. 15, 5437–5453, 2009.
- C. Garcia-Cervera, J. Lu, Y. Xuan and W. E, “A Linear Scaling Subspace Iteration Algorithm with Optimally Localized Non-Orthogonal Wave Functions for Kohn-Sham Density Functional Theory,” *Phys. Rev. B*, vol. 79, no. 11, pp. 115110–115110-13, 2009.
- L. Lin, J. Lu, L. Ying, R. Car and W. E, “Fast algorithm for extracting the diagonal of the inverse matrix with application to the electronic structure analysis of metallic systems,” *Comm. Math. Sci.*, vol. 7, pp. 755–777, 2009.
- T. Li, J. Liu and W. E, “A probabilistic framework for network partition,” *Phys. Rev. E*, vol. 80, no. 2, pp. 026106–026106-12, 2009.
- L. Lin, J. Lu, R. Car and W. E, “Multipole representation of the Fermi operator with application to electronic structure analysis of metallic systems,” *Phys. Rev. B*, vol. 79, no. 11, pp. 115133–115133-10, 2009.
- W. E and D. Li, “On the crystallization of 2D hexagonal lattices,” *Comm. Math. Phys.*, vol. 286, no. 3, pp. 1099–1140, 2009.
- G. Wu, G. Lu, C.J. Garcia-Cervera, and W. E, “Density-gradient-corrected embedded atom method,” *Phys. Rev. B*, vol. 79, pp. 035124–035124-8, 2009.
- W. Gao and W. E “Orbital minimization with localization,” *Discrete and Continuous Dynamical Systems*, vol. 23, no. 1-2, pp. 249–264, 2009.
- W. E and D. Li, “The Andersen thermostat in molecular dynamics,” *Comm. Pure Appl. Math.*, vol. 61, pp. 96–136, 2008.
- C.J. García-Cervera, W. Ren, J. Lu, and W. E, “Sequential multiscale modeling using sparse representation,” *Comm. Comp. Phys.*, vol. 4, no. 5, pp. 1025–1033, 2008.
- X. Zhou, W. Ren and W. E, “Adaptive minimum action method for the study of rare events,” *J. Chem. Phys.*, vol. 128, pp. 104111, 2008.
- W. E, T. Li and E. Vanden-Eijnden, “Optimal partition and effective dynamics of complex networks,” *Proc. Natl. Acad. Sci. USA*, vol. 105, pp. 7907–7912, 2008.
- T. Li, A. Abdulle and W. E, “Effectiveness of implicit methods for stiff stochastic differential equations,” *Comm. Comput. Phys.*, vol. 3, no. 2, pp. 295–307, 2008.
- W. E, “Multiscale analysis of density functional theory,” Proceedings of the 6th International Congress of Industrial and Applied Mathematics, Zurich, 2007.
- C. J. Garcia-Cervera, J. Lu and W. E, “Asymptotics-based sublinear scaling algorithms and applications to the study of electronic structure of materials,” *Comm. Math. Sc.*, vol. 5, pp. 990–1026, 2007.

- S. Chen, W. E, Y. Liu and C.-W. Shu, “A discontinuous Galerkin implementation of a domain decomposition method for kinetic-hydrodynamic coupling multiscale problems in gas dynamics and device simulations,” *J. Comput. Phys.*, vol. 225, no. 2, pp. 1314–1330, 2007.
- W. E, B. Engquist, X. Li, W. Ren and E. Vanden-Eijnden “Heterogeneous multiscale methods: A review,” *Comm. Comput. Phys.*, vol. 2, no. 3, pp. 367–450, 2007.
- W. E and J. Lu, “Seamless multiscale modeling via dynamics on fiber bundles,” *Comm. Math. Sci.*, vol. 5, no. 3, pp. 649–663, 2007.
- W. E and J. Lu, “The continuum limit and QM-continuum approximation of quantum mechanical models of solids,” *Comm. Math. Sci.*, vol. 5, no. 3, pp. 679–696, 2007.
- W. E and J. Lu, “The Elastic Continuum Limit of the Tight Binding Model,” *Chinese Ann. Math. Ser. B*, vol. 28, no. 6, pp. 665–676, 2007.
- W. E, D. Liu and E. Vanden-Eijnden, “Nested stochastic simulation algorithms for chemical kinetic systems with multiple time scales,” *J. Comput. Phys.*, vol. 221, no. 1, pp. 158–180, 2007.
- W. E and P.B. Ming, “Cauchy-Born Rule and the Stability of Crystalline Solids: Dynamic Problems,” *Acta Math. Appl. Sin. Engl. Ser.*, vol. 23, no. 4, pp. 529–550, 2007.
- W. E and P.B. Ming, “Cauchy-Born Rule and the Stability of Crystalline Solids: Static Problems,” *Arch. Rat. Mech. Anal.*, vol. 183, no. 2, pp. 241–297, 2007.
- W. E, W. Ren, E. Vanden-Eijnden, “Simplified and improved string method for computing the minimum energy paths in barrier-crossing events,” *J. Chem. Phys.*, vol. 126, no. 16, pp. 164103–164103-8, 2007.
- W. Guo, T. P. Schulze and W. E, “Simulation of impurity diffusion in a strained nanowire using off-lattice KMC,” *Comm. Comput. Phys.*, vol. 2, no. 1, pp. 164–176, 2007.
- D. Hu, P. Zhang and W. E, “Continuum theory of a moving membrane,” *Phys. Rev. E*, vol. 75, no. 4, pp. 041605–041605-11, 2007.
- X. Li and W. E, “Variational boundary conditions for molecular dynamics simulations of crystalline solids at finite temperature: Treatment of the thermal bath,” *Phys. Rev. B*, vol 76, no. 10, pp. 104107–104107-22, 2007.
- C.B. Muratov, E. Vanden-Eijnden, W. E, “Noise can play an organizing role for the recurrent dynamics in excitable media,” *Proc. Natl. Acad. Sci.*, vol. 104, no. 3, pp. 702–707, 2007.
- T. Qian, W. Ren, J. Shi, W. E and P. Sheng, “Numerical study of metastability due to tunneling: The quantum string method,” *Phys. A*, vol. 379, no. 2, pp. 491–502, 2007.
- W. Ren and W. E, “Boundary conditions for the moving contact line problem,” *Phys. Fluids*, vol. 19, pp. 022101–022101-15, 2007.
- X. Yue and W. E, “The local microscale problem in the multiscale modeling of strongly heterogeneous media: Effect of boundary conditions and cell size,” *J. Comput. Phys.*, vol. 222, no. 2, pp. 556–572, 2007.

- W. E and P. Zhang, “A molecular theory of inhomogeneous liquid crystal flow and the small Deborah number limit,” *Methods Appl. Anal.*, vol. 13, no. 2, pp. 181–198, 2006.
- W. E and E. Vanden-Eijnden, “Towards a theory of transition paths,” *J. Stat. Phys.*, vol. 123, no. 3, pp. 503–523, 2006.
- X.-T. Li and W. E, “Variational boundary conditions for molecular dynamics simulation of solids at low temperature,” *Comm. Comput. Phys.*, vol. 1, no. 1, pp. 135–175, 2006.
- X.-P. Wang, K. Wang and W. E, “Simulations of 3-D Domain Wall Structures in Thin Films,” *Discrete Contin. Dyn. Syst. Ser. B*, vol. 6, no. 2, pp. 373–389, 2006.
- J.Z. Yang and W. E, “Generalized Cauchy-Born rules for elastic deformation of sheets, plates, and rods: Derivation of continuum models from atomistic models,” *Phys. Rev. B*, vol. 74, no 18, pp. 184110–184110-11, 2006.
- D. Zhou, P. Zhang and W. E, “Modified models of polymer phase separation,” *Phys. Rev. E*, vol. 73, no. 6, pp. 061801–061801-11, 2006.
- S. Chen, W. E and C.-W. Shu, “The Heterogeneous Multiscale Method Based on the Discontinuous Galerkin Method for Hyperbolic and Parabolic Problems,” *Multiscale Model. Simul.*, vol. 3, no. 4, pp. 871–894, 2005.
- N. Choly, G. Lu, W. E and E. Kaxiras, “Multiscale Simulations in Simple Metals: A Density-Functional Based Methodology,” *Phys. Rev. B*, vol. 71, no. 9, pp. 094101–094101-16, 2005.
- W. E and B. Engquist, “The Heterogeneous Multi-Scale Method for Homogenization Problems,” *Multiscale Methods in Sci. and Eng.*, pp. 89–110, *Lect. Notes in Comput. Sci. Eng.*, vol. 44, Springer, Berlin, 2005.
- X.-T. Li and W. E, “Multiscale Modeling of the Dynamics of Solids at Finite Temperature,” *J. Mech. Phys. Solids*, vol. 53, pp. 1650–1685, 2005.
- W. E and X.-T. Li, “Multiscale Modeling of Crystalline Solids,” *Handbook of Materials Modeling*, vol. A, pp. 1491–1506, Springer Netherlands, 2005.
- W. E, D. Liu and E. Vanden-Eijnden, “Nested stochastic simulation algorithm for chemical kinetic systems with disparate rates,” *J. Chem. Phys.*, vol. 123, no. 19, pp. 194107–194107-8, 2005.
- W. E, D. Liu and E. Vanden-Eijnden, “Analysis of multiscale methods for stochastic differential equations,” *Comm. Pure Appl. Math.*, vol. 58, no. 11, pp. 1544–1585, 2005.
- W. E and P.B. Ming, “Analysis of the local quasicontinuum method,” *Frontiers and Prospects of Contemp. Appl. Math.*, pp. 18–32, *Contemporary Appl. Math.*, vol. 6, Higher Education Press, Beijing, 2005.
- W. E, P.B. Ming and P.-W. Zhang, “Analysis of the heterogeneous multiscale method for elliptic homogenization problems,” *J. Amer. Math. Soc.*, vol. 18, no. 1, pp. 121–156, 2005.
- W. E, W. Ren and E. Vanden-Eijnden, “Finite temperature string method for the study of rare events,” *J. Phys. Chem. B*, vol. 109, no. 14, pp. 6688–6693, 2005.
- W. E, W. Ren and E. Vanden-Eijnden, “Transition pathways in complex systems: Reaction coordinates, iso-committor surfaces and transition tubes,” *Chem. Phys. Lett.*, vol. 143, no. 1-3, pp. 242–247, 2005.

- W. Ren, E. Vanden-Eijnden, P. Maragakis and W. E, "Transition pathways in complex systems: Application of the finite temperature string method to the alanine dipeptide," *J. Chem. Phys.*, vol. 123, no. 13, pp. 134109–134109-12, 2005.
- C.B. Muratov, E. Vanden-Eijnden and W. E, "Self-induced stochastic resonance in excitable systems," *Phys. D*, vol. 210, no. 3-4, pp. 227–240, 2005.
- W. Ren and W. E, "Heterogeneous multiscale method for the modeling of complex fluids and micro-fluidics," *J. Comput. Phys.*, vol. 204, no. 1, pp. 1–26, 2005.
- S. Succi, W. E and E. Kaxiras, "Lattice Boltzmann Methods for Multiscale Fluid Problems," *Handbook of Materials Modeling*, Part B, pp. 2475–2486, Springer Netherlands, 2005.
- X. Yue and W. E, "Numerical methods for multiscale transport equations and application to two-phase porous media flow," *J. Comput. Phys.*, vol. 210, no. 2, pp. 656–675, 2005.
- W. E and B. Engquist, "The heterogeneous multiscale method. Second Intl. Congress of Chinese Mathematicians," *Proc. of ICCM2001*, Taipei, pp. 57–74, New Studies in Advanced Mathematics, vol. 4, Intl. Press, 2004.
- T.-J. Li, E. Vanden-Eijnden, P.W. Zhang and W. E, "Stochastic models of polymeric fluids at small Deborah number," *J. Non-Newtonian Fluid Mechanics*, vol. 121, no. 2-3, pp. 117–125, 2004.
- W. E and X.-T. Li, "Analysis of the Heterogeneous Multiscale Method for Gas Dynamics," *Methods Appl. Anal.*, vol. 11, no. 4, pp. 557–572, 2004.
- W. E, X. Li, E. Vanden-Eijnden "Some Recent Progress in Multiscale Modeling," *Multiscale Modelling and Simulation*, pp. 3–22, *Lect Notes Comput. Sci. Eng.*, vol. 39, Springer, Berlin, 2004.
- W. E, T.-J. Li and P.-W. Zhang, "Well-posedness for the dumbbell model of polymeric fluids," *Comm. Math. Phys.*, vol. 248, no. 2, pp. 409–427, 2004.
- W. E and P.B. Ming, "Analysis of multiscale methods," *J. Comput. Math.*, vol. 22, no. 2, pp. 210–219, 2004.
- W. E, W. Ren, E. Vanden-Eijnden, "Minimum action method for the study of rare events," *Comm. Pure Appl. Math.*, vol. 57, no. 5, pp. 637–656, 2004.
- W. E and E. Vanden-Eijnden, "Metastability, conformation dynamics, and transition pathways in complex systems," *Multiscale modelling and simulation*, pp. 35–68, *Lect. Notes Comput. Sci. Eng.*, vol. 39, Springer, Berlin, 2004.
- W. E and X. Yue, "Heterogeneous multiscale method for locally self-similar problems," *Comm. Math. Sci.*, vol. 2, no. 1, pp. 137–144, 2004.
- X. Nie, S. Chen, W. E and M.O. Robbins, "A Continuum and Molecular Dynamics Hybrid Method for Micro- and Nano-Fluid Flow," *J. Fluid Mech.*, vol. 500, pp. 55–64, 2004.
- X. Nie, S. Chen, W. E and M. Robbins, "Hybrid continuum-atomistic simulation of singular corner flow," *Phys. Fluids*, vol. 16, no. 10, pp. 3579–3591, 2004.

- Y. Xiang and W. E, “Misfit elastic energy and a continuum model for epitaxial growth with elasticity on vicinal surfaces,” *Phys. Rev. B*, vol. 69, no. 3, pp. 035409–035409-16, 2004.
- Y. Xiang, D.J. Srolovitz, L.-T. Cheng and W. E, “Level set simulations of dislocation-particle bypass mechanisms,” *Acta Materialia*, vol. 52, no. 7, pp. 1745–1760, 2004.
- A. Abdulle and W. E, “Finite difference heterogeneous multi-scale method for homogenization problems,” *J. Comput. Phys.*, vol. 191, no. 1 pp. 18–39, 2003.
- L.-T. Cheng and W. E, “The heterogeneous multi-scale method for interface dynamics,” Recent advances in scientific computing and partial differential equations (Hong Kong, 2002), pp. 43–53, *Contemp. Math.*, vol. 330, Amer. Math. Soc., Providence, RI, 2003.
- W. E, “Analysis of the heterogeneous multiscale method for ordinary differential equations,” *Comm. Math. Sci.*, vol. 1, no. 3, pp. 423–436, 2003.
- W. E and B. Engquist, “The heterogeneous multiscale methods,” *Comm. Math. Sci.*, vol. 1, no. 1, pp. 87–132, 2003.
- W. E and B. Engquist, “Multiscale Modeling and Computation,” *Notices Amer. Math. Soc.*, vol. 50, no. 9, pp. 1062–1070, 2003.
- W. E, B. Engquist and Z. Huang, “Heterogeneous multiscale method: A general methodology for multiscale modeling,” *Phys. Rev. B*, vol. 67, no. 9, pp. 092101–092101-4, 2003.
- W. E and J.-G. Liu, “Gauge method for viscous incompressible flows,” *Comm. Math. Sci.*, vol. 1, no. 2, pp. 317–332, 2003.
- W. E, W. Ren and E. Vanden-Eijnden, “Energy landscape and thermally activated switching of submicron-sized ferromagnetic elements,” *J. Appl. Phys.*, vol. 93, no. 4, pp. 2275–2282, 2003.
- W. E and E. Vanden-Eijnden, “A note on generalized flows,” *Phys. D*, vol. 183, no. 3-4, pp. 159–174, 2003.
- C.J. Garcia-Cervera and W. E, “Improved Gauss-Seidel projection method for micromagnetics simulations,” *IEEE Trans. Magnetics*, vol. 39, no. 3, pp. 1766–1770, 2003.
- C.J. Garcia-Cervera, Z. Gimbutas and W. E, “Accurate numerical methods for micromagnetics simulations with general geometries,” *J. Comput. Phys.*, vol. 184, no. 1, pp. 37–52, 2003.
- T. Schulze, P. Smereka and W. E, “Coupling kinetic Monte-Carlo and continuum models with application to epitaxial growth,” *J. Comput. Phys.*, vol. 189, no. 1, pp. 197–211, 2003.
- Y. Xiang, L.-T. Cheng, D.J. Srolovitz and W. E, “A level set method for dislocation dynamics,” *Acta Materialia*, vol. 51, no. 18, pp. 5499–5518, 2003.
- W. E and Z. Huang, “A dynamic atomistic-continuum method for the simulation of crystalline materials,” *J. Comput. Phys.*, vol. 182, no. 1, pp. 234–261, 2002.
- W. E and D. Liu, “Gibbsian dynamics and invariant measures for stochastic dissipative PDEs,” *J. Stat. Phys.*, vol. 108, no. 5-6, pp. 1125–1156, 2002.

- W. E and J.-G. Liu, "Projection method III: spatial discretization on the staggered grid," *Math. Comp.*, vol. 71, no. 237, pp. 27–47, 2002.
- W. E, T.-J. Li, P.-W. Zhang, "Convergence of a stochastic method for the modeling of polymeric fluids," *Acta Math. Appl. Sin. Engl. Ser.*, vol. 18, no. 4, pp. 529–536, 2002.
- W. E, W. Ren and E. Vanden-Eijnden, "String method for the study of rare events," *Phys. Rev. B*, vol. 66, no. 5, pp. 052301–052301-4, 2002.
- W. E, W. Ren and E. Vanden-Eijnden, "Energy Landscapes and Rare Events," *ICM Report*, vol. 1, pp. 621–630, Higher Ed. Press, Beijing, 2002.
- C.B. Muratov and W. E, "Theory of phase separation kinetics in polymer-liquid crystal systems," *J. Chem. Phys.*, vol. 116, no. 11, pp. 4723–4734, 2002.
- P. Palffy-Muhoray, T. Kosa and W. E, "Brownian motors in the photoalignment of liquid crystals," *Appl. Phys. A*, vol. 75, no. 2, pp. 293–300, 2002.
- P. Palffy-Muhoray, T. Kosa, W. E, "Brownian ratchets and the photoalignment of liquid crystals," *Braz. J. Phys.*, vol.32 no.2b, pp. 552–563, São Paulo, 2002.
- P. Palffy-Muhoray, T. Kosa and W. E, "Dynamics of a Light Driven Molecular Motor," *Mol. Cryst. Liq. Cryst.*, vol. 375, no. 1, pp. 577–592, 2002.
- Q. Wang, W. E, C. Liu, P.-W. Zhang, "Kinetic theory for flows of nonhomogeneous rodlike liquid crystalline polymers with a nonlocal intermolecular potential," *Phys. Rev. E*, vol. 65, no. 5, pp. 051504–051504-7, 2002.
- Y. Xiang and W. E, "Nonlinear evolution equation for the stress-driven morphological instability," *J. Appl. Phys.*, vol. 91, no. 11, pp. 9414–9422, 2002.
- W. E, "Stochastic hydrodynamics," *Current Developments in Mathematics*, pp. 109–147, Intl. Press, Somerville, MA, 2001.
- W. E, "Stochastic PDES in turbulence theory," Proc. 1st Intl. Congress Chinese Math. (Beijing, 1998), pp. 27–46, *AMS/IP Stud. Adv. Math*, vol. 20, Amer. Math. Soc., Providence, RI, 2001.
- W. E, "Selected Problems in Materials Science," *Mathematics Unlimited - 2001 and Beyond*, Engquist, Björn; Schmid, Wilfried (Eds.), pp. 407–432, Springer, Berlin, 2001.
- W. E, "Numerical methods for viscous incompressible flows: some recent advances," *Advances in scientific computing*, p. 29, Science Press, 2001.
- W. E and Z. Huang, "Matching conditions in atomistic-continuum modeling of materials," *Phys. Rev. Lett.*, vol. 87, no. 13, pp. 135501–135501-4, 2001.
- W. E and J.C. Mattingly, "Ergodicity for the Navier-Stokes equation with degenerate random forcing: Finite-dimensional approximation," *Comm. Pure Appl. Math.*, vol. 54, no. 11, pp. 1386–1402, 2001.
- W. E, J.C. Mattingly and Ya. Sinai, "Gibbsian dynamics and ergodicity for the stochastically forced Navier-Stokes equation," *Comm. Math. Phys.*, vol. 224, no. 1, pp. 83–106, 2001.

- W. E and E. Vanden-Eijnden, “Turbulent Prandtl number effect on passive scalar advection,” *Phys. D*, vol. 152-153, pp. 636–645, 2001.
- W. E and N.K. Yip, “Continuum theory of epitaxial crystal growth. I,” *J. Stat. Phys.*, vol. 104, no. 1-2, pp. 221–253, 2001.
- C.J. Garcia-Cervera and W. E, “Effective dynamics for ferromagnetic thin films,” *J. Appl. Phys.*, vol. 90, no. 1, pp. 370–374, 2001.
- J.-G. Liu and W. E, “Simple finite element method in vorticity formulation for incompressible flows,” *Math. Comp.*, vol. 70, no. 234, pp. 579–593, 2001.
- M.I. Mendeleev, D.J. Srolovitz and W. E, “Grain-boundary migration in the presence of diffusing impurities: simulations and analytical models,” *Philos. Mag. A*, vol. 81, no. 9, pp. 2243–2269, 2001.
- T. Schulze and W. E, “A continuum model for the growth of epitaxial films,” *J. Crystal Growth*, vol. 222, no. 1-2, pp. 414–425, 2001.
- X.-P. Wang, C.J. Garcia-Cervera and W. E, “A Gauss-Seidel projection method for micromagnetics simulations,” *J. Comput. Phys.*, vol. 171, no. 1, pp. 357–372, 2001.
- W. E, “Boundary layer theory and the zero-viscosity limit of the Navier-Stokes equation,” *Acta Math. Sin.*, vol. 16, no. 2, pp. 207–218, 2000.
- W. E, K. Khanin, A. Mazel and Ya. Sinai, “Invariant measures for Burgers equation with stochastic forcing,” *Ann. of Math.*, vol. 151, no. 3, pp. 877–960, 2000.
- W. E and J.-G. Liu, “Gauge finite element method for incompressible flows,” *Intl. J. Numer. Methods in Fluids*, vol. 34, no. 8, pp. 701–710, 2000.
- W. E and Ya. Sinai, “Recent results on mathematical and statistical hydrodynamics,” *Russ. Math. Survey*, vol. 55, no. 4, pp. 635–666, 2000.
- W. E and E. Vanden-Eijnden, “Statistical theory for the stochastic Burgers equation in the inviscid limit,” *Comm. Pure Appl. Math.*, vol. 53, no. 7, pp. 852–901, 2000.
- W. E and E. Vanden-Eijnden, “Another note on forced Burgers turbulence,” *Phys. Fluids*, vol. 12, no. 1, pp. 149–154, 2000.
- W. E and E. Vanden-Eijnden, “Generalized flows, intrinsic stochasticity and turbulent transport,” *Proc. Natl. Acad. Sci.*, vol. 97, no. 15, pp. 8200–8205, 2000.
- W. E and X.-P. Wang, “Numerical methods for the Landau-Lifshitz equation,” *SIAM J. Numer. Anal.*, vol. 38, no. 5, pp. 1647–1665, 2000.
- W. E and N.K. Yip, “Continuum limits of step flow models,” *Intl. Conf. Differential Equations*, vol. 1, 2 (Berlin, 1999), pp. 448–453, World Sci. Publishing, River Edge, NJ, 2000.
- T. Kosa, W. E and P. Palffy-Muhoray, “Brownian motors in the photoalignment of liquid crystals,” *Intl J. Eng. Sci.*, vol. 38, no. 9-10, pp. 1077–1084, 2000.

- H. Yuan, W. E and P. Palffy-Muhoray, “Analytical solution of Maxwell’s equations in lossy and optically active crystals,” *Phys. Rev. E*, vol. 61, no. 3, pp. 3264–3266, 2000.
- R. Caffisch, W. E, M. Gyure, B. Merriman and C. Ratsch, “Kinetic model for a step edge in epitaxial growth,” *Phys. Rev. E*, vol. 59, no. 6, pp. 6879–6887, 1999.
- W. E, “Aubry-Mather theory and periodic solutions of the forced Burgers equation,” *Comm. Pure Appl. Math.*, vol. 52, no. 7, pp. 811–828, 1999.
- W. E and P. Palffy-Muhoray, “Dynamics of filaments during the isotropic-smectic A phase transition,” *J. Nonlin. Sci.*, vol. 9, pp. 417–437, 1999.
- W. E and E. Vanden-Eijnden, “On the statistical solution of the Riemann equation and its implications for Burgers turbulence,” *Phys. Fluids*, vol. 11, no. 8, pp. 2149–2153, 1999.
- W. E and E. Vanden-Eijnden, “Asymptotic theory for the probability density functions in Burgers turbulence,” *Phys. Rev. Lett.*, vol. 83, no. 13, pp. 2572–2575, 1999.
- W. E and P. Palffy-Muhoray, “Orientational ratchets and angular momentum balance in the Janossy effect,” *Mol. Cryst. Liq. Cryst.*, vol. 320, no. 1, pp. 193–206, 1998.
- W. E and P. Palffy-Muhoray, “Domain size in the presence of random fields,” *Phys. Rev. E*, vol. 57, no. 1, pp. 135–137, 1998.
- W. E, “Nonlinear continuum theory of smectic-A liquid crystals,” *Arch. Rat. Mech. Anal.*, vol. 137, no. 2, pp. 159–175, 1997.
- W. E and B. Engquist, “Blowup of solutions of the unsteady Prandtl’s equation,” *Comm. Pure Appl. Math.*, vol. 50, no. 12, pp. 1287–1293, 1997.
- W. E, K. Khanin, A. Mazel and Ya. Sinai, “Probability distribution functions for the random forced Burgers equation,” *Phys. Rev. Lett.*, vol. 78, no. 10, pp. 1904–1907, 1997.
- W. E and J.-G. Liu, “Finite difference schemes for incompressible flows in the velocity-impulse density formulation,” *J. Comput. Phys.*, vol. 130, no. 1, pp. 67–76, 1997.
- W. E and J.-G. Liu, “Finite difference methods for 3D viscous incompressible flows in the vorticity-vector potential formulation on nonstaggered grids,” *J. Comput. Phys.*, vol. 138, no. 1, pp. 57–82, 1997.
- W. E and P. Palffy-Muhoray, “Phase separation in incompressible systems,” *Phys. Rev. E*, vol. 55, no. 4, pp. R3844–R3846, 1997.
- F. Otto and W. E, “Thermodynamically driven incompressible fluid mixtures,” *J. Chem. Phys.*, vol. 107, no. 23, pp. 10177–10177-8, 1997.
- W. E, “Dynamics of vortices in superconductors,” *World Congress of Nonlinear Analysts ’92*, vol. 1-4 (Tampa, FL, 1992), pp. 3811–3821, de Gruyter, Berlin, 1996.
- W. E and J.-G. Liu, “Vorticity boundary condition and related issues for finite difference schemes,” *J. Comput. Phys.*, vol. 124, no. 2, pp. 368–382, 1996.

- W. E and J.-G. Liu, “Essentially Compact Schemes for Unsteady Viscous Incompressible Flows,” *J. Comput. Phys.*, vol. 126, no. 1, pp. 122–138, 1996.
- W. E and J.-G. Liu, “Projection Method II: Godunov-Ryabenki Analysis,” *SIAM J. Numer. Anal.*, vol. 33, no. 4, pp. 1597–1621, 1996.
- W. E and J.-G. Liu, “Finite difference schemes for incompressible flows in vorticity formulations,” Vortex flows and related numerical methods, II (Montreal, PQ, 1995), pp. 181–195, *ESAIM Proc.*, vol. 1, Soc. Math. Appl. Indust., Paris, 1996.
- W. E, Yu. Rykov and Ya. Sinai, “Generalized variational principles, global weak solutions and behavior with random initial data for systems of conservation laws arising in adhesion particle dynamics,” *Comm. Math. Phys.*, vol. 177, no. 2, pp. 349–380, 1996.
- M. Avellaneda and W. E, “Statistical properties of shocks in Burgers turbulence,” *Comm. Math. Phys.*, vol. 172, no. 1, pp. 13–38, 1995.
- M. Avellaneda, R. Ryan and W. E, “PDFs for velocity and velocity gradients in Burgers’ turbulence,” *Phys. Fluids*, vol. 7, no. 12, pp. 3067–3071, 1995.
- W. E and J.-G. Liu, “Projection Method I: Convergence and Numerical Boundary Layers,” *SIAM J. Numer. Anal.*, vol. 32, no. 4, pp. 1017–1057, August, 1995.
- Z.-T. Chen and W. E, “Convergence of Legendre Methods for Navier-Stokes Equations,” *J. Comput. Math.*, vol. 12, no. 4, pp. 298–311, 1994.
- P. Constantin, W. E and E.S. Titi, “Onsager’s conjecture on the energy conservation for solutions of Euler’s equation,” *Comm. Math. Phys.*, vol. 165, no. 1, pp. 207–209, 1994.
- W. E, “Dynamics of vortices in Ginzburg-Landau theories with applications to superconductivity,” *Phys. D*, vol. 77, no. 4, pp. 383–404, 1994.
- W. E, “Dynamics of vortex liquids in Ginzburg-Landau theories with applications to superconductivity,” *Phys. Rev. B*, vol. 50, no. 2, pp. 1126–1135, 1994.
- W. E and C.-W. Shu, “A Numerical Resolution Study of High Order Essentially Non-oscillatory Schemes Applied to Incompressible Flow,” *J. Comput. Phys.*, vol. 110, no. 1, pp. 39–46, 1994.
- W. E and C.-W. Shu, “Small-scale structures in Boussinesq convection,” *Phys. Fluids*, vol. 6, no. 1, pp. 49–58, 1994.
- W. E, “Convergence of Fourier Methods for the Navier-Stokes Equations,” *SIAM J. Numer. Anal.*, vol. 30, no. 3, pp. 650–674, 1993.
- B. Engquist and W. E, “Large time behavior and homogenization of solutions of two-dimensional conservation laws,” *Comm. Pure Appl. Math.*, vol. 46, no. 1, pp. 1–26, 1993.
- W. E and C.-W. Shu, “Effective equations and the inverse cascade theory for Kolmogorov flows,” *Phys. Fluids A*, vol. 5, no. 4, pp. 998–1010, 1993.

- Z. Cai and W. E, "Hierarchical method for elliptic problems using wavelet," *Comm. Appl. Numer. Methods*, vol. 8, no 11, pp. 819–825, 1992.
- W. E, "Propagation of oscillations in the solutions of 1-D compressible fluid equations," *Comm. Partial Differential Equations*, vol. 17, no. 3-4, pp. 347–370, 1992.
- W. E, "Homogenization of linear and nonlinear transport equations," *Comm. Pure Appl. Math.*, vol. 45, no. 3, pp. 301–326, 1992.
- W. E, "Homogenization of scalar conservation laws with oscillatory forcing terms," *SIAM J. Appl. Math.*, vol. 52, no. 4, pp. 959–972, 1992.
- W. E, "Convergence of Spectral Methods for Burgers' Equation," *SIAM J. Numer. Anal.*, vol. 29, no. 6, pp. 1520–1541, 1992.
- W. E and D. Serre, "Correctors for the homogenization of conservation laws with oscillatory forcing terms," *Asymptotic Anal.*, vol. 5, no. 4, pp. 311–316, 1992.
- T.F. Chan, W. E and J. Sun, "Domain decomposition interface preconditioners for fourth-order elliptic problems," *Appl. Numer. Math.*, vol. 8, no 4-5, pp. 317–331, 1991.
- W. E, "A class of homogenization problems in the calculus of variations," *Comm. Pure Appl. Math.*, vol. 44, no. 7, pp. 733–759, 1991.
- W. E and R.V. Kohn, "The initial value problem for measure-valued solutions of a canonical 2 x 2 system with linearly degenerate fields," *Comm. Pure Appl. Math.*, vol. 44, no. 8-9, pp. 981–1000, 1991.
- W. E and H. Yang, "Numerical study of oscillatory solutions of the gas-dynamic equations," *Stud. Appl. Math.*, vol. 85, no. 1, pp. 29–52, 1991.
- W. E and T.Y. Hou, "Homogenization and convergence of the vortex method for 2-D Euler equations with oscillatory vorticity fields," *Comm. Pure Appl. Math.*, vol. 43, no. 7, pp. 821–855, 1990.
- W. E, M. Mu and H.C. Huang, "A posteriori error estimates in finite element methods," *Chinese Quart. J. Math.*, (Chinese) vol. 3, no. 1, pp. 97–107, 1988.
- W. E, H.C. Huang and W. Han, "Error analysis of local refinements of polygonal domains," *J. Comput. Math.*, vol. 5, no. 1, pp. 89–94, 1987.
- H.C. Huang and W. E, "A posteriori error estimates for finite element methods for one-dimensional boundary value problems," *Chinese Quart. J. Math.*, (Chinese) vol. 2, no. 1, pp. 43–47, 1987.
- H.C. Huang, W. E and M. Mu, "Extrapolation combined with multigrid method for solving finite element equations," *J. Comput. Math.*, vol. 4, no. 4, pp. 362–367, 1986.
- W. E, "The optimal parameters of the AOR method and their effect," *Math. Numer. Sin.*, (Chinese) vol. 6, no. 3, pp. 329–333, 1984.