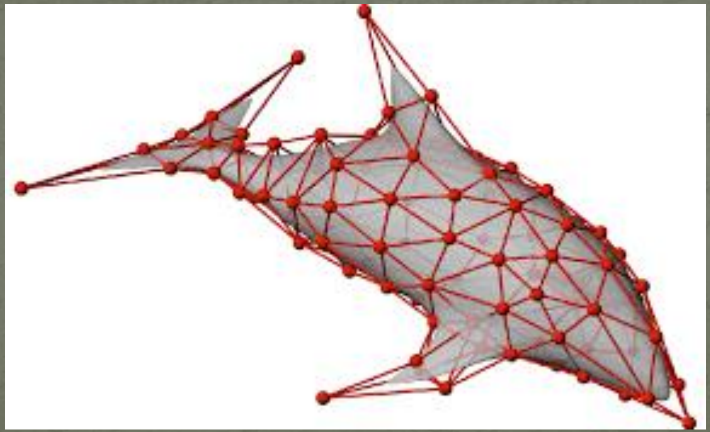
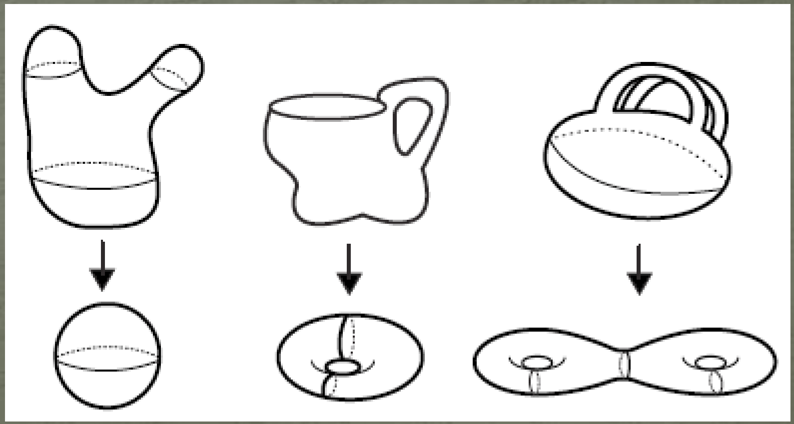
# Graphs, planes & surfaces

- **Context:** Testing whether given network is **planar** (can be drawn in the plane, without crossing edges)
- **Practical Motivation:** Hard problems are often easier for planar graphs
- **General Goal:** Find algorithms for surfaces besides the plane?





# Surface embedding: Goal factoring

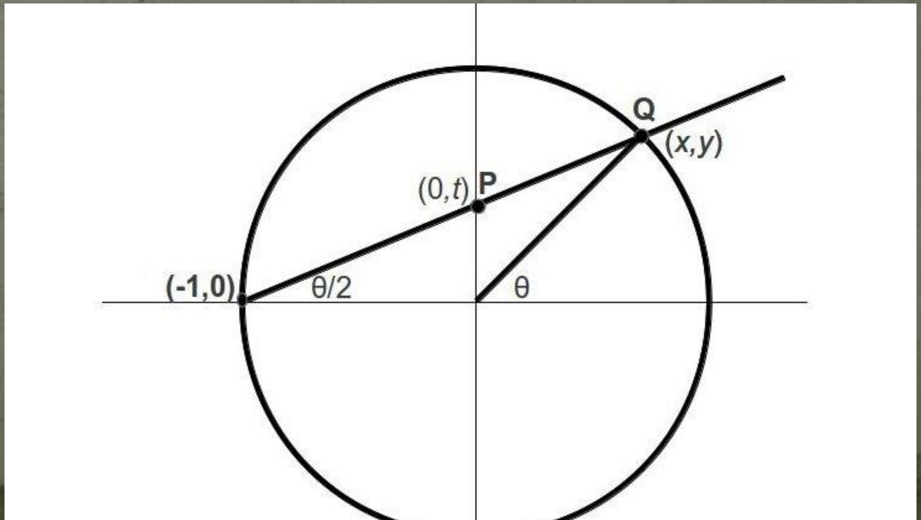
- Race with two independent legs (either would be interesting on its own)
- Baton 2 (R&D infrastructure, build steroids)
  - Hanani-Tutte Conjecture: If an **even drawing** exists, then in fact, an **embedded drawing** must exist too
  - To spell it out: if you give me a picture where every two (independent) edges cross an **even number of times**, then we know indirectly that God's album actually has a picture where every two edges cross **zero times**
- Baton 1 (would the steroids be useful?)
  - System of equations in bits (0/1, even/odd)

# Baton 1: Algebraic geometry

- Baton 1 (would the steroids be useful?)
  - System of equations in bits (0/1, even/odd)
- How fast can we solve the system?
  - For the plane/sphere, the system is linear
    - High school algebra (Gaussian elimination), just with lots of variables
  - For coffee mugs (tori) and other surfaces, the system is quadratic, which are generally hard
    - Can we make sense of this specific quadratic system? (Efficient underlying structure?)
    - Or is it like Republican health care? (Lifetime to digest?)

# Example: Quadratic, but easy

- Is a circle genuinely quadratic, or is that a disguise?
- Depends on context, but an algebraic geometer might say a circle can be **parameterized** by a linear scale
- Start with a point, like (-1,0), on the unit circle.
- Draw line through that point, (-1,0), with slope t.
- As t varies, you cover every point on the circle exactly once.
- Bonus: t = 1/2 gives you (x,y) = (3/5, 4/5).
- Fractions t give all Pythagorean triples!



# Baton 2: Algebraic topology

- Baton 2 (R&D infrastructure, build steroids)
  - Hanani-Tutte Conjecture: If an **even drawing** exists, then in fact, an **embedded drawing** must exist too
- This is kind of mysterious: embedded (crossing-free) drawings are easy to visualize, but even (crossing-even) drawings are not
- We're trying to understand this more fundamentally in terms of algebraic topology (specifically **obstructions to embedding**)

# Thanks for your time!

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  - GIANT, CNRS & GIPSA-lab
  - Profs. Arnaud de Mesmay & Francis Lazarus
  - MIT-France
  - https://thatsmaths.com/2014/01/23/pythagorean-triples/ for the circle picture