Permanence conditions [EGAIV$_2$, §7]. Consider a property $P$ of noetherian local rings. Denote by $\varphi: (A, m) \to (B, n)$ a local flat map of noetherian local rings.

(I) (Ascent) If $B \otimes_A \kappa(p)$ is geometrically $P$ over $\kappa(p)$ for every prime ideal $p \subseteq A$, and if $A$ is $P$, then $B$ is $P$.

(II) (Descent) If $B$ is $P$, then $A$ is $P$.

(III) (Lifting) If there exists a nonzerodivisor $t \in A$ such that $A/t$ is $P$, then $A$ is $P$.

(IV) (Localization) If $A$ is $P$, then $A_p$ is $P$ for every prime ideal $p \subseteq A$.

Theorem 1 [Mur, Thm. A(ii)]. Let $P$ be a property of noetherian local rings such that regular implies $P$, and such that $P$ satisfies (I)–(IV). Consider a closed flat morphism $f: Y \to X$ of locally noetherian schemes, where for all $x \in X$, the fibers of $O_{X,x} \to \hat{O}_{X,x}$ are geometrically $P$ over $\kappa(x)$. Then, the following locus is stable under generization:

$$U_P(f) := \{x \in X \mid f^{-1}(x) \text{ is geometrically } P \text{ over } \kappa(x)\}$$


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