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A Three-Prong Standard for Practical Evaluation of Partisan Gerrymandering

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ABSTRACT. Since the United States Supreme Court's *Davis v. Bandemer* ruling of 1986, partisan gerrymandering for statewide electoral advantage has been held to be justiciable. The existing Supreme Court standard, culminating in *Vieth v. Jubelirer*, holds that a test for gerrymandering should demonstrate both intents and effects, and that partisan gerrymandering is recognizable by its asymmetry: for a given distribution of popular votes, if the parties switch places in popular vote, the numbers of seats will change in an unequal fashion. However, the asymmetry standard is only a broad statement of principle, and no analytical method for assessing asymmetry has yet been held by the Supreme Court to be manageable. This article proposes a three-prong statistical standard to reliably assess asymmetry in state-level districting schemes: (a) unrepresentative distortion beyond normal statistical variation based on nationwide composition of districts; (b) unexpectedly few close losses by the redistricting party; and (c) a discrepancy between the district-by-district mean and median vote share. These three tests give consistent results with one another and can largely be carried out using pencil, paper, and a hand calculator, without examination of either maps or redistricting procedures. I apply this standard to a variety of districting schemes, starting from the original "Gerry-mander" of 1812 and including modern cases. In post-2010 Congressional elections, partisan gerrymandering in a handful of states generated effects that are larger than the total nationwide effect of population clustering. Arizona legislative districts, the object of the upcoming *Harris v. Arizona Independent Redistricting Commission* case, fail to meet any component of the three-prong standard. I propose that this effects-based standard may be robust enough to supplant the need to demonstrate intent. The three-prong statistical standard offered here adds to the judge's toolkit for rapidly and rigorously identifying the effects of redistricting.

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I. INTRODUCTION

A. LEGAL STANDARDS GOVERNING REDISTRICTING IN A SINGLE-MEMBER-DISTRICT SYSTEM

The US system of representative democracy contains at its core a tension based on the fact that all federal and many state and local legislators are elected in single-member districts. In such a system, citizens are assigned to districts in which they vote, and elect a single legislator[1]. In a cardinal advantage of this system, every citizen is represented by a specific legislator in the House of Representatives or a lower-level legislative chamber[2]. In this way, citizens have a personal, unique, and direct path for seeking redress of government-related issues.

Interposed in this seemingly straightforward path between citizens and legislators is the process by which districts are drawn. District maps are redrawn anew following each decade's Census, which determines the distribution of Representatives in the House of Representatives among the states[3]. Given its number of representatives, each state has the responsibility to draw up[4] U.S. House and state legislative districts, a process that is constrained by natural variations in where people live, laws that govern the drawing of boundaries, compromises during the

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[1] ANDREW HACKER, CONGRESSIONAL DISTRICTING (1964); Tory Mast, *The History of Single Member Districts for Congress*, FAIR VOTE (Aug. 20, 2015, 1:50 PM), http://archive.fairvote.org/?page=526. Number of Congressional Districts; number of Representatives from each District, Pub. L. No. 90-196, 5 Stat. 491, 12 Stat. 572, 17 Stat. 28, 22 Stat. 5, 31 Stat. 733, 37 Stat. 13 (2000) (codified at 2 U.S. Code § 2c).

[2] Similar problems exist at the level of state legislatures. The analysis described in this article is also applicable to evaluating state-level districting results.

[3] *U.S. Const. art. I, § 2.*

[4] *U.S. Const. art. I, § 4.* See also *State-by-State Redistricting Procedures*, BALLOTPEDIA, http://ballotpedia.org/wiki/index.php/State-by-state_redistricting_procedures (last visited Aug. 20, 2015).

legislative process, and whether voting laws applied by the Justice Department and courts allow a set of boundaries to stand. Virtually all districting schemes resulting from this process result in the consequence that representation ends up not being directly proportional to public support. This consequence is well-known and results from the winner-take-all nature of individual elections[5]. Still, despite this difficulty and the seemingly rickety nature of the districting process, at a national level the party that receives more votes usually receives the majority of seats[6].

In the cases when districting plans are challenged, litigants often assert that the districting process has distorted the ability of voters to elect representatives that fairly reflect their views. A common form of this claim is that of partisan gerrymandering, *i.e.* statewide redistricting efforts that are intended to confer specific advantage to one political party at the expense of another, so that the overall districting scheme elects delegations that do not fairly reflect the state's overall proportion of voters. Nonpartisan Congressional scholars have identified recent rises in partisan gerrymandering in the United States as a substantial risk to representative democracy[7]. Partisan gerrymandering has formed the basis of many court challenges to redistricting, including challenges in several states since the 2010 Census[8].

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[5] See Edward R. Tufte, *The Relationship Between Votes and Seats in Two-Party Systems*, 67 AMERICAN POLITICAL SCIENCE REVIEW 540 (1973). For example, in a two-party system, it is theoretically possible for one political party to win 49% of the vote in every district, yet not win a single delegate. Although such an extreme case is highly improbable, strong deviations from proportionality are nevertheless an inherent risk of a winner-take-all district system. From a democratic standpoint, a central question is how to avoid the most extreme distortions. Actual outcomes are considerably less distorted than the extreme hypothetical scenario described above.

[6] Sam Wang, *The Great Gerrymander of 2012*, N.Y. TIMES, Feb. 2, 2013, at SR1.

[7] THOMAS E. MANN & NORMAN J. ORNSTEIN, IT'S EVEN WORSE THAN IT LOOKS: HOW THE AMERICAN CONSTITUTIONAL SYSTEM COLLIDED WITH THE NEW POLITICS OF EXTREMISM (2012); Thomas E. Mann & Norman J. Ornstein, *Let's Just Say it: The Republicans are the Problem*, WASH. POST. OPINION BLOG (Apr. 27, 2012), https://www.washingtonpost.com/opinions/lets-just-say-it-the-republicans-are-the-problem/2012/04/27/gIQAxCVUlT_story.html.

[8] A current list of redistricting challenges pending before the Supreme Court can be found at ALL ABOUT REDISTRICTING, http://redistricting.lls.edu/cases.php#sct (last visited August 27, 2015).

Partisan gerrymandering may be challenged on two Constitutional grounds: equal protection and the "one person, one vote" principle, and First Amendment protection of speech. Justice Anthony Kennedy has noted that the First Amendment can be interpreted as a mandate for "not burdening or penalizing citizens because of their participation in the electoral process, their voting history, their association with a political party, or their expression of political views[9]. Under general First Amendment principles those burdens in other contexts are unconstitutional absent a compelling government interest."[10]

In 1986, the Supreme Court established in *Davis v. Bandemer*[11] that partisan gerrymandering is deemed justiciable, *i.e.* suitable for judicial review. The Court recognized a cause of action based on a two-prong test: the intent to create a legislative districting map to disempower the voters for one party, and the effect that an election based on that map led to a distorted outcome.

Since that time, a central difficulty has been establishing a standard for the *Bandemer* test that is manageable, *i.e.* that gives a reliable and usable result. In *Bandemer*, the justices described the effects prong in general terms. Justice White advocated an analysis of an entire districting plan: "A statewide challenge, by contrast, would involve an analysis of "the voters' direct or indirect influence on the elections of the state legislature as a whole," while also acknowledging that this was "of necessity a difficult inquiry."[12] In the *Vieth v. Jubelirer* case in 2004[13], which concerned gerrymandering in Pennsylvania, the plurality opinion, signed by four justices, stated

[9] *Vieth*, 541 U.S. at 314 (J. Kennedy, concurring); *Elrod v. Burns*, 427 U. S. 347, 362 (1976).

[10] *Elrod*, id., at 362.

[11] *Davis v. Bandemer*, 478 U.S. 109 (1986).

[12] *Bandemer*, 478 U.S. at 143.

[13] *Vieth v. Jubelirer*, 541 U.S. 267 (2004).

that no acceptable standard had been established in the intervening 18 years, and therefore that it was time to abandon the search. However, in a separate concurrence, Justice Anthony M. Kennedy provided a fifth vote against a finding of chicanery in Pennsylvania, but left the door open for future remedies in other cases if a clear standard could be established. The dissenting four justices voted in favor of a finding of partisan gerrymandering. The *LULAC v. Perry case* in 2004 addressed mid-decade redistricting in Texas, but without altering the state of play as established by *Vieth*. In this article I use statistical methods to construct a three-prong test for symmetry that is quantitatively-based, *Vieth*-compatible, and potentially usable as a manageable standard.

B. NEW GERRYMANDERING THREATS IN MODERN TIMES

Partisan gerrymandering is quite old, dating to the establishment of Pennsylvania's assembly districts in 1705[14]. When the term "Gerry-mander" was coined in 1812[15], it was used to mock one specific, oddly-shaped district encompassing northern parts of Essex County. However, the broader target of opprobrium was the overall goal of gaining more seats at the statewide level. Redistricters from Governor Elbridge Gerry's Democratic-Republican party sought to take popular support that was closely divided between their party and the other major party, the Federalists, and divide it among districts to favor their own side. The stratagem worked: Federalists won the two-party vote share by a margin of 51%-to-49% over the Democratic-Republicans, but ended up severely outnumbered in the General Court, with only 11 seats to the Democratic-Republicans' 29 seats. This was achieved by packing of Federalists so

[14] ELMER C. GRIFFITH, THE RISE AND DEVELOPMENT OF THE GERRYMANDER (1907).

[15] *The Gerrymander: A New Species of Monster*, BOSTON GAZETTE (Mar. 26, 1812), http://www.loc.gov/exhibits/treasures/trr113.html.

that they won an average of 71%-to-29% of the two-party vote in the districts they carried. Democratic-Republicans were arranged to win by smaller margins, averaging 56%-to-44% per district[16].

Direct evidence for a partisan gerrymander's success is the consequent distortion of an election result. However, such a distortion does not necessarily persist over time. In the case of the original Gerry-mander, the next election, in 1813, showed a rapid reversal of fortune for the Democratic-Republicans. Public anger led to increased Federalist turnout and a 56%-to-44% popular-vote victory, with an outcome of 29 Senate seats to the Democratic-Republicans' 11. This mirror-perfect reversed outcome was achieved by only a 5% increase in the Federalists' vote share. Such a dramatic swing was made possible by the fact that Democratic-Republican-learning districts were engineered to deliver extremely narrow victories, so that a small swing in opinion was sufficient to influence many races.

The example of 1812-1813 shows that a partisan gerrymander's effects can be reversed if sentiments change sufficiently. A gerrymander can also weaken if voters physically change their residence. When district boundaries are carefully constructed based on the pattern of voter residence at a single point in time, it is more likely than not that voter mobility will tend to dissipate the gerrymandered advantage, much as a child's carefully built sandcastle, once left unattended, will erode with the wind.

Technological advances have opened the possibility of drawing more sophisticated gerrymanders that can potentially lead to more secure and lasting advantages for the party in charge of redistricting. Several factors come into play:

[16] *Lampi Collection of American Electoral Returns, 1787–1825. American Antiquarian Society, 2016.*

1) Redistricting was once done on a county-by-county basis. However, detailed Census and voter-registration information is now available on a block-by-block basis. Districting software, in both commercial and freely available varieties, allows the use of this information to create extremely detailed boundaries that can separate populations of voters from one another with exquisite spatial resolution. Professionals use proprietary software[17] to draw districts, but free software like Dave's Redistricting App[18] allows activists and ordinary citizens alike to enter the fray.

2) Voters themselves have tended to cluster into Democratic- and Republican-preferring communities. Generally speaking, Democratic voters are most often found at high percentages in regions of higher population density, and Republican voters in regions of lower population density, and these tendencies have intensified in recent years[19] as part of a phenomenon that has been termed the Big Sort[20]. This sorting leads voters to become aggregated into easy-to-handle contiguous chunks, and opens the possibility that redistricting can be more reliable as a neighborhood's partisan tendencies become more stable.

Overall, reliable partisan voting and the Big Sort create geographic patterns that make it easier to gerrymander. In this way, polarization can facilitate the ease with which gerrymandering is done[21]. Conversely, any increase in safe seats created by gerrymandering also

[17] *Maptitude for Redistricting*, CALIPER, http://www.caliper.com/mtredist.htm (last visited Aug. 20, 2015).

[18] DAVE'S REDISTRICTING, http://gardow.com/davebradlee/redistricting/launchapp.html (last visited Aug. 20, 2015).

[19] Wendy K. Tam Cho, James G. Gimpel, and Iris S. Hui, *Voter Migration and the Geographic Sorting of the American Electorate,* 103 ANNALS OF THE ASSOCIATION OF AMERICAN GEOGRAPHERS 856 (2013); Jesse Sussell, *New Support for the Big Sort Hypothesis: An Assessment of Partisan Geographic Sorting in California, 1992-2010*, 46 PS, POLITICAL SCIENCE & POLITICS 768 (2013).

[20] BILL BISHOP, THE BIG SORT: WHY THE CLUSTERING OF LIKE-MINDED AMERICA IS TEARING US APART (2009).

[21] The converse belief is common, i.e. the belief that gerrymandering of districts leads to increased polarization. However, polarization of voters and legislators is not reduced in cases where district boundaries do not matter, such

increases the number of districts in which representatives are insulated from changes in voter sentiment.

Based on analysis in the 1990s, the effects of partisan Congressional gerrymanders have been estimated to last on average for a few election cycles[22]. Changes in technical tools and population clustering, as well as a greater awareness of the advantages of aggressive districting[23], open the possibility that gerrymandered districts may be more durable now than they would have been even ten years ago. An increasing number of state governments have come under one-party rule[24], and partisan gerrymandering has reached recent extremes of asymmetry[25]. All these factors working together – the Big Sort, more detailed data, computer-based districting, and single-party rule – represent ways by which gerrymandering may exert more influence and undermine the principle of representative democracy. These factors magnify the need for a manageable standard to define partisan gerrymandering.

C. SEARCHING FOR A MANAGEABLE STANDARD: THE CURRENT STATE OF PLAY

as the Senate, at-large House districts, or in randomly drawn districts. Thus gerrymandering appears not to be a direct cause of polarization. See Michael J. Barber and Nolan McCarty, *Causes and Consequences of Polarization,* in SOLUTIONS TO POLITICAL POLARIZATION IN AMERICA 15, 27-28 (Nathaniel Persily ed., 2015).

[22] Andrew Gelman & Gary King, *Enhancing Democracy Through Legislative Redistricting*, 88 AMERICAN POLITICAL SCIENCE REVIEW 541 (1994).

[23] Gregory Giroux, *Republicans Win Congress as Democrats Get Most Votes*, BLOOMBERG BUSINESS (Mar. 18, 2013 8:00 PM), http://www.bloomberg.com/news/articles/2013-03-19/republicans-win-congress-as-democrats-get-most-votes.

[24] Carl E. Klarner, *State Partisan Balance Data, 1937 – 2011*, 2013, IQSS DATAVERSE NETWORK V1, http://hdl.handle.net/1902.1/20403 (last visited Aug. 21, 2015) (updating Carl E. Klarner, *Measurement of the Partisan Balance of State Government*, 3 STATE POLITICS AND POLICY QUARTERLY 309 (2003)); U.S. CENSUS BUREAU, STATISTICAL ABSTRACT OF THE UNITED STATES 261 TABLE 418 (2012).

[25] Nicholas Stephanopoulos & Eric McGhee, *Partisan Gerrymandering and the Efficiency Gap*, 82 UNIV. OF CHICAGO L. REV. 831 (2015).

In *Vieth v. Jubelirer*, Justice Anthony M. Kennedy stated in his separate concurrence: "When presented with a claim of injury from partisan gerrymandering, courts confront two obstacles. First is the lack of comprehensive and neutral principles for drawing electoral boundaries. No substantive definition of fairness in districting seems to command general assent. Second is the absence of rules to limit and confine judicial intervention."[26] This concern has been longstanding. In *Bandemer*, Justice O'Connor expressed concern that the plurality's standard "will over time either prove unmanageable and arbitrary or else evolve towards some loose form of proportionality."[27] This statement was quoted in the *Vieth* plurality decision[28] by Justice Scalia, who also expressed pessimism that such standards could be established.

The three-prong test in this article addresses these concerns. However, considering the multiple foregoing criticisms, it is worth reviewing some previous candidate criteria for partisan gerrymandering that were offered in *Vieth v. Jubelirer* and *LULAC v. Perry,* but which the Supreme Court rejected or did not fully embrace in its decisions. When closely examined, those decisions point toward criteria for an acceptable test.

1) Majority of votes, majority of seats. In the *Vieth* case, part 2 of the appellants' effects standard suggested that the "'totality of circumstances' confirms that the map can thwart the plaintiffs' ability to translate a majority of votes into a majority of seats."[29] This standard was described by Justice Breyer in his dissent as the "unjustified use of political factors to entrench a minority in power".

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[26] *Vieth*, 541 U.S. at 306-7 (J. Kennedy, concurring).

[27] *Bandemer*, 478 U.S. at 155 (C.J. Burger, concurring).

[28] *Vieth*, 541 U.S. at 281.

[29] Brief for Appellants at 20, Vieth v. Jubelirer, 541 U.S. 267 (2004) (No. 02–1580).

However, a "majority-majority" standard is vulnerable to variation and chance. As Justice Scalia has written, "In any winner-take-all district system, there can be no guarantee, no matter how the district lines are drawn, that a majority of party votes statewide will produce a majority of seats for that party."[30] Although this hypothetical concern is literally true, it elides the possibility that a mathematical analysis can offer clarification. To put these concerns into quantitative terms: the "majority-majority" standard does not take into account the possibility that an outcome could arise not via skulduggery, but by more innocent variations in voting patterns or district-drawing. The majority-majority standard can potentially be improved in two ways: by (a) identifying a "no guarantee" zone of naturally-likely election outcomes in which Scalia's objection might plausibly apply, and outside of which the objection does not apply; and (b) generalizing such a standard to other popular-vote outcomes. I will address these issues using statistical analysis to identify no-guarantee zones of ambiguity.

2) Characteristics of individual districts. Justice Souter suggested[31] that partisan gerrymandering could be identified by examining individual districts. However, Justice Scalia wrote that "the central problem is determining when political gerrymandering has gone too far." Such a problem is intrinsically difficult, because partisan gerrymandering arises from patterns of districting, not single districts. Indeed, a given set of boundaries for one district might or might not lead to an overall partisan advantage, depending on how the other districts are drawn.

Legislators have long sought (a) to protect individual incumbents, and (b) to maximize the advantage for their party. These two goals are not perfectly consonant, and indeed lean on different methods. Indeed, what is good for an individual incumbent is not always good for his or

[30] *Vieth*, 541 U.S. at 289.

[31] *Vieth*, 541 U.S. at 296.

her party at the statewide level, and *vice versa*. Because of this, an important distinction must be made between single-district and statewide gerrymandering[32]. In brief, a single-district gerrymander eliminates competition in only one race, while statewide gerrymandering consists of an artful pattern of many single-district gerrymanders to distort the overall outcome.

In single-district gerrymandering, the core technique is to draw a single district's boundaries to increase one's number of supporters. However, self-interest does not necessarily lead to anti-proportional outcomes. Indeed, although incumbent protection reduces competition in individual districts, it can still achieve majoritarian representation. As an example, imagine if incumbents of both parties agree to draw all districts to have a similar advantage, resulting in districts that split 60%-40% in either direction. In such a circumstance, the party with greater popular support must necessarily win more seats.[33] Although incumbent protection is a self-serving act by legislators, it is constitutionally accepted[34] and when it happens symmetrically, interests can still be represented. In summary, although it seems inimical to democracy to make individual districts less competitive, this act by itself is neither inherently anti-majoritarian, nor is it justiciable by current standards.

Consistent with this point, the *Vieth* decision ruled out the presence of circuitous boundaries as an indicator of partisan gerrymandering. Two reasons support this view. First,

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[32] Even more broadly, the word "gerrymander" is colloquially used to describe a range of partisan offenses, including polarization of voters. Such overbroad usage dates back at least a hundred years. *See* GRIFFITH, *supra* note 14. In this article the term is restricted to the stricter sense of using district boundaries to obtain an advantage for a candidate, faction, or party.

[33] Mathematically, this can be stated as follows. If party A gets fraction V of the total two-party vote, and all districts on both sides will be split 60-40, then F, the fraction of A-favoring districts, must satisfy $0.6F+0.4(1-F)=V$. If furthermore $V>0.5$, i.e. party A wins the popular vote, then $F>0.5$, i.e. the number of A-favoring districts must also be a majority. This principle is generally true, and is limited only by the fact that for a finite number of districts, the margins of the individual districts would not be precisely 60-40.

[34] *Vieth*, 541 U.S. at 298; *Bush v. Vera*, 517 U. S. 952, 1047–1048 (1996) (J. Souter, dissenting).

circuitous boundaries can be drawn for non-partisan reasons, for instance to unify communities of interest, or to create districts of near-identical population, or to construct a district with a large number of minority-group voters, the "majority-minority" districts drawn under Section 2 of the Voting Rights Act. Perhaps as a consequence of these various criteria, circuitousness of boundaries has risen since the 1960s[35]. Conversely, relatively straight boundaries do not guarantee a majoritarian outcome: in Michigan, where many Congressional district boundaries follow straight north-south and east-west lines for miles at a time, the House popular vote was 53.2% Democratic, 46.8% Republican in 2012, and 50.9% Democratic, 49.1% Republican in 2014, in both cases leading to a delegation of 5 Democrats and 9 Republicans. Second, by their nature, gerrymandered districts of opposing political parties must adjoin one another, so that any circuitous boundary belongs to multiple districts, often controlled by opposing political parties. In summary, boundaries can serve as an indicator of partisan problems in districting, but are difficult to use as a sole criterion, and do not reveal whether a systemic statewide problem exists. I will therefore eschew the shapes of districts in constructing statistical tests.

3) Consideration of districting procedures. In *Bandemer*, Justice Powell proposed to identify "whether district boundaries had been drawn solely for partisan ends to the exclusion of 'all other neutral factors relevant to the fairness of redistricting.'"[36] This wording by Powell suggests that it might be possible to detect gerrymandering by comparing the procedures used with more neutral procedures, drawing hypothetical districts, and comparing the predicted hypothetical outcomes with actual election results.

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[35] Stephen Ansolabehere and Maxwell Palmer, A Two Hundred-Year Statistical History of the Gerrymander, Presentation at the Congress & History Conference, Vanderbilt University (May 22-23, 2015).

[36] *Bandemer*, 478 U.S. at 161 (opinion concurring in part and dissenting in part); *see also Vieth*, 541 U.S. at 164–165.

However, the plurality in *Vieth* criticized the examination of procedures as being excessively vague. Examination of procedures presents a judge with the question of whether a hypothetical alternative plan was drawn with partisan intent. But whenever a district map is drawn, some decisions must inevitably be made about whether, and how, to join or split communities. Districting seeks to pursue many goals, including "contiguity of districts, compactness of districts, observance of the lines of political subdivision, protection of incumbents of all parties, cohesion of natural racial and ethnic neighborhoods, compliance with requirements of the Voting Rights Act of 1965 regarding racial distribution, etc."[37] In addition to these goals, which explicitly serve the public good, legislators and political parties also serve their own interests. Doubtless the complexity of such a complex process leads to the "difficult inquiry" cited by Justice White.

In one recent example, Chen and Rodden[38] have developed a sophisticated, automated procedure in which a computer program draws districts "in a random, partisan-blind manner, using only the traditional districting criteria of equal apportionment and geographic contiguity and compactness of single-member legislative districts."[39] However, their computerized procedure explicitly omits concerns that might emerge during the legislative process. For example, why, in a densely populated area, should a boundary be as straight as it is in a sparsely

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[37] *Vieth*, 541 U.S. at 284.

[38] Jowei Chen and Jonathan Rodden, *Cutting Through the Thicket: Redistricting Simulations and the Detection of Partisan Gerrymanders*, 14 ELECTION LAW JOURNAL (forthcoming 2015) [hereinafter Chen & Rodden, *Cutting Through the Thicket*]; Jowei Chen and Jonathan Rodden, *Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures*, 8 QUARTERLY JOURNAL OF POLITICAL SCIENCE 239 (2013) [hereinafter Chen & Rodden, *Unintentional Gerrymandering*]; Jowei Chen and Jonathan Rodden, Report on Computer Simulations of Florida Congressional Districting Plans (February 15, 2013) (unpublished manuscript) (on file with author) [hereinafter Chen & Rodden, Report on Computer Simulations]; Jowei Chen and Jonathan Rodden, Supplemental Report on Partisan Bias in Florida's Congressional Redistricting Plan (October 21, 2013) (unpublished manuscript) (on file with author) [hereinafter Chen & Rodden, Supplemental Report].

[39] Chen & Rodden, *Unintentional Gerrymandering*, *supra* note 38 at 248.

populated area? I choose to describe this automated procedure not as a negative criticism of it, but simply to point out that consideration of districting procedures leads to a proliferation of choices and value judgments – in short, political questions. In this way the problem of judging has split like the heads of the Hydra, making the problem harder to manage.

The broader point is that when districts are drawn at random, different definitions produce different results. Even if one were to use a set of rules (contiguous and compact districts, keeping communities intact, and so on) to simulate all possible districts, that would only identify the sample space of all possibilities. It would not identify the probability or desirability of different types of outcomes in practice.

As an alternative to simulations of the districting process, I suggest that it might be better to use real election results. Election results nationwide contain a rich source of actual legislative dealmaking. In my approach for establishing a manageable standard, I assume that national House districts constitute a sample that reflects accepted standards of districting practice, following a wide variety of geographic, demographic, political, and legal constraints to produce districts of varying partisan composition. In other words, the great give and take of the legislative process in all 50 states has performed a natural experiment, in which a wide range of prevalent districting standards, measured in terms of outcomes, has been established. For this reason I will use nationwide election results as a baseline for the Analysis #1 and #2.

4) Predicting partisan loyalties using minor statewide races. Because voters often vote according to their partisan loyalties, it has been suggested that overall voter sentiment can be gauged by examining low-profile statewide races such as secretary of state or attorney general, where candidate-specific factors are ostensibly minimized. However, the *Vieth* plurality stated[40]

[40] *Vieth*, 541 U.S. at 286.

that this standard is not judicially manageable. In evidence pertaining to the *Vieth* case, in the 2000 Pennsylvania election some Republicans won statewide and some Democrats won; these races thus did not provide unambiguous guidance on overall partisan preference. This concern suggests that House elections would be the best source of guidance about partisan intention. Given the skepticism surrounding the use of information from other races, the most manageable standard appears to require use of the results of House elections themselves.

5) Partisan symmetry. As a guiding principle to defining fairness in districting, Grofman and King have suggested[41] partisan symmetry: the idea that if the popular vote were reversed, the seat outcome should also reverse. This work was cited by multiple opinions in *LULAC*, and appears to be generally acceptable. Districting schemes are often tested by detailed procedures such as the JudgeIt algorithm, which has been used by its inventors and other researchers[42] with great success to analyze individual districts. Like Chen and Rodden's automated procedures for district map-drawing, JudgeIt contains technical assumptions which do not necessarily capture the entirety of the legislative process. However, neither JudgeIt or automated map-drawing have yet led to the enunciation of a standard that the Supreme Court has found to be manageable.

Indeed, claims of partisan gerrymandering have largely failed. In the words of the four-vote *Vieth* plurality, the application of the *Bandemer* standard "has almost invariably produced the same result (except for the incurring of attorney's fees) as would have obtained in the

[41] Bernard Grofman & Gary King, *The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering after* LULAC v. Perry, 6 ELECTION LAW JOURNAL 2 (2007).

[42] S.C. McKee, J.M. Teigen, and M. Turgeon, *The Partisan Impact of Congressional Redistricting: The Case of Texas, 2001-2003, 87* SOCIAL SCIENCE QUARTERLY 308; P. Gronke & J.M. Wilson, *Competing Redistricting Plans as Evidence of Political Motives - The North Carolina Case, 27* AMERICAN POLITICS QUARTERLY 147.

question were nonjusticiable: judicial intervention has been refused."[43] The *Vieth* plurality further stated that "...no judicially discernible and manageable standards for adjudicating political gerrymandering claims have emerged. Lacking them, we must conclude that political gerrymandering claims are nonjusticiable and that *Bandemer* was wrongly decided."[44] In other words, unless a manageable standard can be found, partisan gerrymandering may soon be considered no longer justiciable, in practice or in fact.

D. DESIRABLE QUALITIES OF A MANAGEABLE STANDARD

In summary, the rejection of the foregoing standards in the *Vieth* decision indicates that a manageable standard must at least have the following minimum qualities: (1) It should recognize a zone of ambiguity. (2) It should apply to a wide range of popular-vote outcomes. (3) It should not use circuitousness of geographic boundaries or districting procedures. (4) It should not use election results for offices other than the ones that are in dispute. Finally, any standard that can be clearly stated without case-specific or mathematics-intensive assumptions might even allow a court to guide experts in their use of mathematical or statistical methods.

E. QUANTITATIVE METHODS CAN IDENTIFY NATIONAL AND STATE-LEVEL IMBALANCES

To establish the three-prong test, it is first necessary to lay out basic principles as a starting point for constructing the analysis. To be representative, a district-based system should tend to produce the following three desirable properties[45]:

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[43] *Vieth*, 541 U.S. at 279.

[44] *Vieth*, 541 U.S. at 281.

[45] To bridge the gap between political science views of good districting and a legal standard, quantitative measures provide a common language that can be analyzed using statistical methods. Mathematical terminology allows the

(1) A greater share of the popular vote should be correlated with a greater share of legislative seats.

(2) For a broad range of starting points, a swing in the popular-vote share changes from one election to the next, leads to a swing in seat share in the same direction. This quality is closely related to the idea that seats should be competitive.

(3) The party that gets more votes also gets more seats.

Although these abstract principles are often not achieved in practice, they represent concepts by which to make an initial diagnosis of how well districting processes are succeeding, democratically speaking. It should be noted that these starting assumptions allow for the fact that single-member district systems inherently do not achieve proportional representation[46]. The three outcomes listed above are desirable because they describe a situation in which popular sentiment is represented dynamically in the legislature. Conversely, deviation from these principles insulates the legislature from voters.

In nationwide elections, majoritarian representativeness is the norm. In the U.S. House of Representatives, when a major party gets more than 50% of the vote, it almost always gets over 50% of the seats (**Figure 1**). In 35 elections, this basic principle has been violated only twice: in

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concepts to be expressed compactly, and sets a template that can be used by later scholars to perform their own calculations. For the three properties listed here, if V is the vote share won by one party and S is the fraction of legislative seats that it wins, the criteria can be stated quantitatively as

(1) and (2) dS/dV is positive (i.e. S's slope relative to V is upward) over most of V's naturally occurring range. This quality is often referred to as "responsiveness". See Bernard Grofman, *Measures of Bias and Proportionality in Seats-Votes Relationships,* 9 POLITICAL METHODOLOGY 295 (1983); Andrew Gelman and Gary King, *Estimating the Electoral Consequences of Legislative Redistricting,* 85 JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION 274 (1990).

(3) S(V) goes through the point (0.5, 0.5). When S(V) does not come near this point, the amount by which it deviates is a measure of "partisan bias"; see GROFMAN AND KING, *supra* note 41.

[46] Tufte, *supra* note 5.

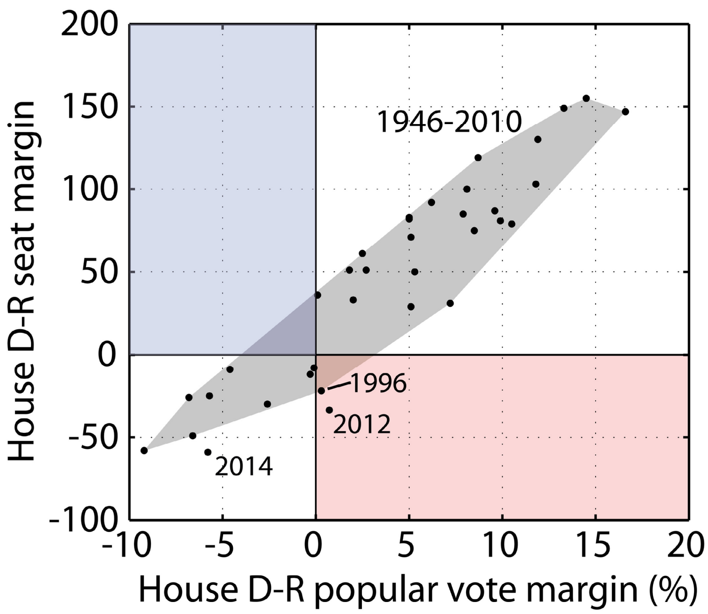

Figure 1: Relationship between seats and votes in the U.S. House of Representatives, 1946-2014. Each point shows one year's nationwide outcome. The gray shaded zone encompasses elections through 2010. The color-shaded zones indicate antimajoritarian outcomes: blue for a Democratic majority won by a Republican-majority popular vote, and pink for a Republican majority won by a Democratic-majority popular vote. Note that both 2012 and 2014 fall outside the gray zone, an indication of a shift in districting conditions from longstanding historical practice. The difference between Democratic and Republican popular vote was calculated defining the sum of the two parties' votes as 100%, i.e. as the two-party vote share.

1996 and in 2012[47]. Thus, for the most part, national House elections meet property (3): the more popular party controls the House.

The plot represents the sum total of elections in 50 states, and by giving an aggregate view of the entire districting process may conceal many sins. Detecting problems in districting requires examination at a state by state level. Thus one can formulate a similar naïve standard at the state level, that the party that wins more than half the votes (out of the top two parties) should get at least half the seats in a state's delegation.

As an example, consider Colorado in 2012. There, 51.4% of the two-party vote went to Republican candidates, and 4 out of 7 representatives were Republicans. Colorado's delegation therefore represented its partisans "fairly," i.e. to meet property (3) above. However, in that same

[47] A failure rate of 2 out of 35, or 6%, may be considered acceptable, when one considers the following comparison: in the history of the United States, the popular vote winner has failed to win the Presidency in 4 out of 57 elections (see DAVE LEIP'S ATLAS OF PRESIDENTIAL ELECTIONS, http://uselectionatlas.org/ (last visited Aug. 20, 2015), a 7% rate. However, Presidential elections rely on fixed state boundaries. Retaining representative performance in legislative elections carries added risk due to changes in where and how district boundaries are drawn.

election, five states failed to clear even this low bar: Arizona, Michigan, North Carolina, Pennsylvania and Wisconsin. In these five states – and in the nation as a whole – the partisan interests of voters are not being represented fairly. Four of these five non-majoritarian outcomes were enabled by their beneficiary, the Republican Party, which controlled the redistricting process[48], and the fifth state, Arizona, was redistricted by a bipartisan commission. Thus antimajoritarian outcomes often, but not always, reflect the partisan interests of those who draw the boundaries.

Anti-majoritarian outcomes do not, by themselves, constitute proof of deliberate distortion of electoral processes. But they do present concrete evidence that the relationship between voting and representative outcomes can be influenced by those who draw the districts. As political parties become a greater predictor of legislative voting patterns[49], representing partisan loyalties accurately becomes increasingly important for getting policy outcomes to reflect popular sentiment.

Even if some imagined ideal of districting could maximize the likelihood of a majoritarian outcome, lack of perfect correspondence can still arise by chance and small variations in opinion. In 2012, if a few thousand voters had cast their ballots for a Republican instead of a Democrat in the 1st or 2nd district of Arizona, the delegation would have been, like the state's popular vote, majority Republican. Conversely, if fewer than four thousand voters in Colorado's 6th Congressional District had voted for a Democrat instead of a Republican, that state's delegation would still have been elected by a Democratic majority vote, but be 4-3 in

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[48] Griff Palmer & Michael Cooper, *How Maps Helped Republicans Keep and Edge in the House*, N.Y. TIMES, Dec. 14, 2012, at A10.

[49] Delia Baldassarri & Andrew Gelman, *Partisans without Constraint: Political Polarization and Trends in American Public Opinion*, 114 AMERICAN JOURNAL OF SOCIOLOGY 408 (2008). *See also* NOLAN MCCARTY, KEITH T. POOLE & HOWARD ROSENTHAL, POLARIZED AMERICA: THE DANCE OF IDEOLOGY AND UNEQUAL RICHES (2008).

favor of Republican legislators. Thus, anti-majoritarian outcomes are not always accurate as an indicator of partisan maneuvering. Furthermore, they are also incomplete because they only address the issue of whether seats or votes are above or below 50%. For example, if a party receives 51% of the vote, receiving 55% or 80% of the seats are both majoritarian, but might be viewed quite differently.

In this light it is not surprising that the U.S. Supreme Court has declined to require that districting schemes lead to majoritarian outcomes[50]. But what degree of inequity is allowable? An approach is necessary that takes into account the natural variation that occurs in districting and elections.

I will use both natural variation and the concept of proportionality as a desirable but idealized goal to build the three-prong standard. My approach allows the user to consider conceptual subtleties, and at the same time obtain unambiguous judgements without need for elaborate computation using methods whose details have either not been widely adopted by political science researchers, and/or found by courts not to be persuasive in the outcome. It is hoped that a more straightforward approach might meet with wide approval and serve as a universal tool to objectively assess claims of partisan gerrymandering.

II. THREE STANDARDS FOR EVALUATING PARTISAN ASYMMETRY

The *Vieth* plurality opinion referred disparagingly to the concept of fairness as "flabby"[51]. Quantitative approaches open the possibility of formulating a more muscular definition. This article will give ways to identify partisan unfairness at the whole-state level, resulting in

[50] *Davis*, 478; *Vieth*, 541.

[51] *Vieth*, 541 U.S. at 281.

proposed standards for partisan gerrymandering that do not require the drawing of hypothetical maps. The analysis in this article will be based initially on computer simulations, which can then be used to design tests that no longer require simulation, and can therefore be applied easily and rapidly. Finally, these tests will be applied to several well-known examples. These tests will then be applied to the 2012 election, which provides many statewide House outcomes for analysis. This approach recalls Justice Kennedy's statement that "new technologies may produce new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties. That would facilitate court efforts to identify and remedy the burdens, with judicial intervention limited by the derived standards."[52]

My proposed standard takes the form of a three-prong test. The three prongs have several advantages. First and foremost, they are simple to test and give unambiguous results. None of the three prongs requires the detailed drawing of maps. Because the tests can be stated with mathematical exactness, they can be used as a manageable standard, giving predictable and sensible results – and unambiguous guidance to legislatures and judges. The tests are based on goals of representative democracy, and are based on election outcomes. Consequently the tests do not require evaluation of intent, and can be used either alone or in conjunction with intent-based criteria.

A. ANALYSIS #1: EU-PROPORTIONALITY

1. PROPORTIONAL REPRESENTATION AS AN ASPIRATIONAL GOAL

Although partisan gerrymandering is considered justiciable, another practice that uses similar technical methods is permitted and even mandated under Section 2 of the Voting Rights

[52] *Id.* at 312-313 (J. Kennedy, concurring).

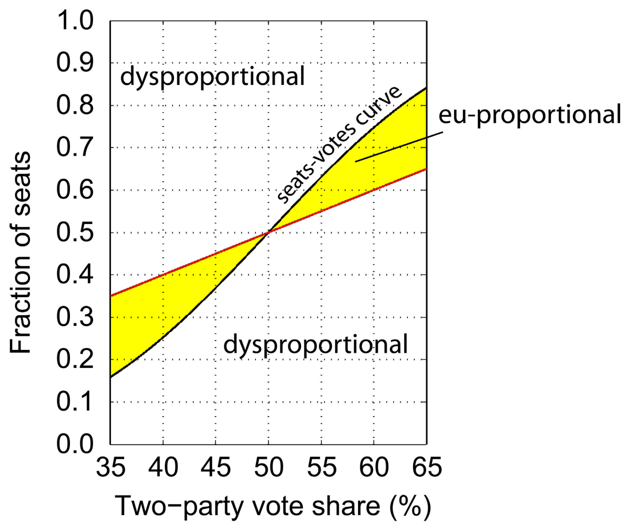

Figure 2: A representation plot for classifying redistricting schemes. The black seats-votes curve indicates the average relationship between seats won (vertical axis) and the popular vote share (horizontal axis), calculated by creating hypothetical delegations using 2012 House district election results. The red straight line indicates proportional representation. Redistricting schemes that fall in the shaded zone between the curve and the line are termed eu-proportional; other outcomes are termed dysproportional. For clarity, the one-sigma zone of ambiguity (see text), which provides an additional criterion, is not shown.

Act: the establishment of districts in which an ethnic minority constitutes a majority of the district's inhabitants. These "majority-minority" districts are constructed to ensure that the interests of identified subgroups are represented. When such minorities are much less than 50% of a state's population, they can end up on the losing side of every election. To counteract this risk, majority-minority districts are constructed to cluster groups with shared interests[53]. Among the standards for the proper establishment of majority-minority districts is the concept that majority-minority districts should comprise a fraction of all districts in proportion to the size of the minority population. This legal standard instantiates a form of proportional representation for ethnic minorities.

Here I extend this principle of proportionality, in a weaker form, to the question of appropriate representation by political parties. I propose a simple extension of the one man, one vote principle: a redistricting plan is acceptable if it moves the seats-to-votes outcome toward proportionality (euproportionality) from prevailing national standards; and unacceptable if it moves the outcome away from proportionality (dysproportionality). This standard can be understood at a glance using a plot (**Figure 2**) that I term a "representation plot," or alternatively

[53] *How New York State's Approved Redistricting Lines Compare with Old Districts*, REDISTRICTING AND YOU, http://www.urbanresearchmaps.org/nyredistricting/map.html (last visited Aug. 20, 2015).

a "bowtie plot," where euproportional outcomes are "inside the bowtie." Since dysproportional outcomes are a major result of partisan gerrymandering, finding such a standard would directly measure gerrymandering's effects[54].

I note that the euproportionality concept specifically does *not* imply the establishment of proportional representation, a rule that is not to be found in the Constitution or in U.S. districting law. Instead, the euproportionality concept relies on the idea that some deviations from an average seats-to-votes relationship are beneficial for representation, whereas other deviations are detrimental. The concept that moving toward proportionality is good encompasses a wide range of concepts that includes (a) establishing appropriate levels of representation for minority groups; (b) allowing the possibility that a political party with less than 50% support might have some enhanced representation relative to what would be predicted from national seats-votes relationships; and (c) setting reasonable limits to how much enhancement from (b) is allowed. In this way, the ideal of moving toward proportionality is simple to state, yet is flexible and contains many permissible outcomes.

Good districting seeks to establish "fair and effective representation for all citizens"[55]. This idealized goal is constrained by the general tendency of single-member districts to achieve disproportionate outcomes rather than proportional representation; in other words, assignment of seats to a party in approximate proportion to its level of popular support. Actual proportional representation is achieved only in systems where it is enforced specifically and directly. For example, in Israel, members of the national legislative body, the Knesset, are assigned so that the

[54] In this plot, the red line indicates proportionality and is a straight line drawn from zero vote share and zero seat fraction to 100% vote share and 100% seat fraction. The seats-votes curve is calculated by resampling to build "fantasy delegations" (see Section II.A.3) and is approximated by the mathematical function that is the area under a bell-shaped curve whose average is 50% vote share, and whose standard deviation is 14% vote share.

[55] *Reynolds v. Sims*, 377 U. S. 533 (1964).

number of a party's seats is proportional to the fraction of its popular vote. Such a system embodies a direct form of the "one man, one vote" principle: each citizen's party preference is reflected proportionally at the national level.

The proportionality concept already exists in court precedent[56], as part of what are called "Gingles criteria" for evaluating districting schemes. As part of the *Gingles* criteria, in *Johnson v. De Grandy*, rough proportionality was identified as a relevant factor, where minority representation is concerned, in evaluating the fairness of a districting plan. Under that standard, the court "hold[s] that no violation of § 2 can be found here, where, in spite of continuing discrimination and racial bloc voting, minority voters form effective voting majorities in a number of districts roughly proportional to the minority voters' respective shares in the voting-age population. While such proportionality is not dispositive in a challenge to single-member districting, it is a relevant fact in the totality of circumstances to be analyzed when determining whether members of a minority group have 'less opportunity than other members of the electorate to participate in the political process and to elect representatives of their choice.'"[57] For example, if a minority group with 20% of a state's eligible population is able to elect representatives in 20% of a state's districts, this argues against violation of conditions as set forth as a consequence of *Thornburg v. Gingles*[58].

Minority groups often support the alternative to the majority group's favored political party, and so if establishment of majority-minority districts under the Voting Rights Act were to approach the limit described in the *Gingles* criteria, the seats-to-votes relationship would move

[56] *Johnson v. De Grandy*, 512 U.S. 997 (1994).

[57] *Id.* at 1000 (finding no violation of §2 of the Voting Rights Act of 1965, 79 Stat. 437, as amended, 42 U. S. C. § 1973).

[58] *Thornburg v. Gingles*, 478 U.S. 30 (1986).

toward proportionality. This concept suggests a natural generalization to other groups such as voters registered to a particular party: if a redistricting plan moves the overall outcome toward proportional representation of political parties, it is desirable, and termed "eu-proportional." This is indicated by the yellow zone in the figure. Conversely, redistricting plans that move outcomes away from proportional representation are termed "dys-proportional."

2. Defining zones of ambiguity

In addition to defining desirable and undesirable directions, a standard for partisan gerrymandering requires a way to determine whether a change could have arisen as part of normal variation in districting as practiced across the United States. I will use the rules of probability to (a) describe that variation, (b) establish what the range of possible outcomes is, and (c) formulate a rule for identifying situations in which a state's new districting scheme has departed sufficiently from normal practice.

To accomplish this analysis, let us consider the concept of "zones of confidence" in which it is possible to state without doubt that a change is dysproportional in favor of a political party. Conversely, situations in which the outcome could have arisen by chance are "zones of ambiguity." To understand this concept, it is helpful to consider a case that is mathematically simple, and does not require computer simulation: equally matched parties.

As pointed out in the plurality opinion in *Vieth v. Jubelirer*, any districting scheme contains the possibility that a majority of votes will lead to a minority of seats. To explore this concern, it is informative to calculate the exact probability that such an advantageous deviation could occur in the absence of intentional partisan districting. The calculation is simplest when the two-party popular-vote share (defined as the fraction of the top two parties' popular vote won by one party) is close to 50% for each party. In this circumstance, party A's seat-share for a random partitioning of N districts is on average N/2, and the probability of party A winning a particular district is 0.5. The actual number of districts won will vary, in the same way that a series of coin tosses are not guaranteed to yield equal numbers of heads and tails. The outcome will be within sigma of the average at least two-thirds of the time. This zone of ambiguity is also known as a "1-sigma range," anywhere within which it an outcome would be fairly unsurprising[59].

To generalize the zone-of-ambiguity calculation, we can use existing districts in the year under examination as a source of information about how vote totals in districts may vary. The inputs to the calculation are the Congressional vote totals for the state under examination, and national district-by-district Congressional results from the same year. This process escapes the burden of drawing boundaries, which requires the researcher to apply his/her standards about "good districting." This calculation yields both a general seats-votes relationship and a statistical confidence interval for the range of outcomes that could be expected in the absence of directed partisan intent. This confidence interval provides an answer to the question of whether a set of election outcomes has deviated sharply from national standards.

[59] For example, if all N races are perfect toss-ups, then they behave like coin tosses, and according to the laws of probability the standard deviation of the outcome, a measure of variation often referred to as "sigma," or σ, is $0.5*\sqrt{N}$. Thus if political parties A and B compete in a state that is composed of 16 Congressional districts, all of which are closely contested, then each party can expect to get 8 seats on average. Sigma for the specific case of all-close-races is $0.5*\sqrt{16} = 2$ seats, and the zone of ambiguity would be 6 to 10 seats for each party. Any outcome within this range could have arisen by chance. It must be noted that the foregoing formula for sigma is a substantial overestimate of real-life situations, because districting generates a mixture of more and less closely-contested districts, and only close contests contribute to uncertainty. To estimate the true value of sigma, which is typically smaller, a more sophisticated approach is required, as detailed in section 3, Computer simulations.

3. Computer simulations of the natural seats-votes curve

To detect dysproportionality by looking only at election returns, computer simulations can be used to ask a simple question: if a given state's popular House vote were split into differently composed districts carved from the same statewide voting population, what would its Congressional delegation look like?

It is possible to calculate each state's appropriate seat breakdown — in other words, how a Congressional delegation would be constituted if its districts were not contorted to protect a political party or an incumbent. This is done by randomly picking combinations of districts from around the United States that add up to the same statewide vote total for each party. Like a fantasy baseball team, a delegation put together this way is not constrained by the limits of geography. On a computer, it is possible to create millions of such unbiased delegations in short order. In this way, one can ask[60] what would happen if a state had districts whose distribution of voting populations was typical of the pattern found in rest of the nation. Because this approach uses existing districts, it uses as a baseline the asymmetries that are present nationwide.[61] Indeed, the average result of these simulations approximates a "natural" seats-votes relationship that can

[60] This can be done by using all 435 House race outcomes. For a state X with N districts, calculate the total popular vote across all N districts. Now pick N races from around the country at random and add up their vote totals. If their vote total matches X's actual popular vote within 0.5%, score it as a comparable simulation. See WANG, *supra* note 6.

[61] It is possible to explore the properties of this simulation procedure by giving it a variety of hypothetical nationwide distributions of districts as starting data. These hypothetical scenarios reveal that the "fantasy delegation" procedure has important features that are required of a descriptor of partisan asymmetry. First, for a symmetric distribution of Congressional districts, *i.e.* a scenario in which Democrat-dominated districts are no more packed than Republican-dominated districts, fantasy delegations are typically majoritarian, awarding more representatives to the party that receives more votes. Second, the fantasy delegations have the same natural variation in partisan composition as the nationwide distribution, as measured by standard deviation. Third, when the nationwide distribution of districts has asymmetry, for instance containing a number of districts that are very packed with one party (as is the case in real life for Democrats), the fantasy delegations show a bias toward the other party, a phenomenon that is well analyzed (reviewed in Chen & Rodden, *Unintentional Gerrymandering*, *supra* note 38).

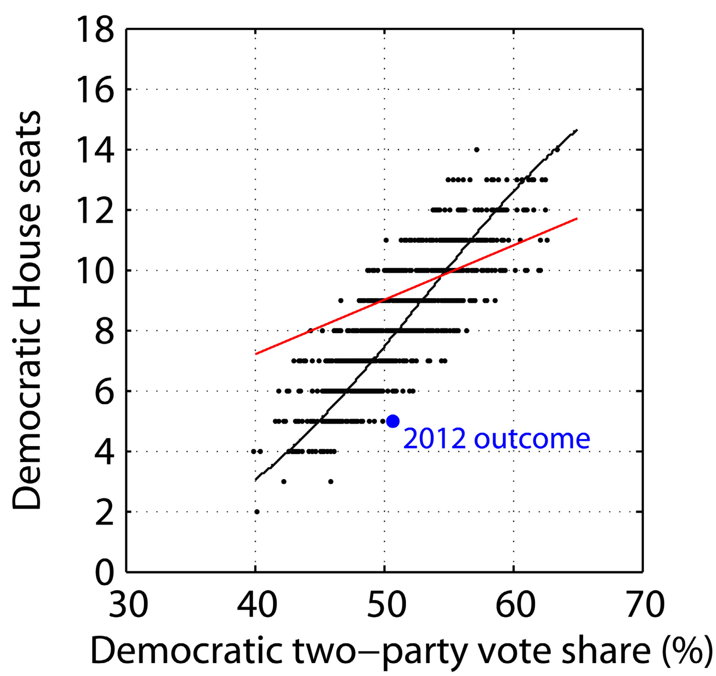

Figure 3: Simulated Pennsylvania House delegations. Each point indicates one hypothetical delegation composed of 18 House districts drawn at random from the national House election of 2012. One thousand simulations are shown. The black curve indicates the average seats-votes curve and the red line indicates proportionality, both as defined in Figure 2. The blue point indicates the actual outcome, which falls in a zone of dysproportionality, "outside the bowtie."

be defined with mathematical rigor and exactitude. In short, these simulations detect distortions in representativeness that are specific to one state, relative to the rest of the nation.

Using a standard ThinkPad X1 Carbon laptop computer equipped with the mathematical program MATLAB, simulation code[62] can perform one million simulations for a state in less than 20 seconds. **Figure 3** shows 1000 such "simulated delegations" for the state of Pennsylvania, along with the actual outcome in blue. The black curve defines a mathematically expected average seats-votes relationship.

It is apparent that most possible redistrictings would have resulted in a more equitable Congressional delegation. For outcomes with the same popular-vote split (50.7% D, 49.3% R), the million simulations gave a median result of 8 Democratic, 10 Republican seats (average, 8.5 D). The actual outcome was 5 Democratic, 13 Republican; however, only 0.2% of the million simulations led to such a lopsided (or a more lopsided) split favoring Republicans.

[62] The MATLAB software is available at GITHUB, https://github.com/ (last visited Aug. 24, 2015).

Pennsylvania is known to have been targeted by the Republican State Legislative Committee's project Redmap, a multiyear effort to facilitate and carry out aggressive redistricting after the 2010 Census.[63] A similar computational analysis of all 50 states can be done to test for additional matches to known cases of partisan redistricting:

For all 50 states, this (**Figure 4**) is calculated using the vote outcomes of non-extreme states (shaded in gray) to feed the simulations. These results coincide strongly with partisan efforts for targeted redistricting[64], and are highly unlikely to have arisen by chance. Red shading indicates Republican Party control over redistricting, blue indicates Democratic Party control, and black indicates nonpartisan commission (AZ, Arizona) or a court-ordered map (TX, Texas). Out of 10 states with

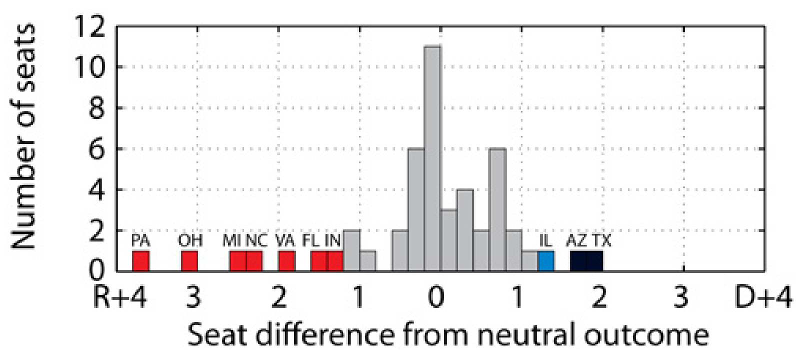

Figure 4: State-by-state differences between simulated and actual outcomes in the 2012 Congressional election. "R+" indicates that the actual outcome was more favorable to Republicans than random resampling from national races. "D+" indicates that the actual outcome was more favorable to Democrats. Color shading for discrepancies greater than 1.2 seats indicates who controlled redistricting: red for Republicans, blue for Democrats, and black for mixed control (AZ, nonpartisan commission, TX Republican Party and court-ordered changes).

[63] Olga Pierce, Justin Elliott & Theodoric Meyer, *How Dark Money Helped Republicans Hold the House and Hurt Voters*, PROPUBLICA (Dec. 21, 2012, 3:36 PM), http://www.propublica.org/article/how-dark-money-helped-republicans-hold-the-house-and-hurt-voters; Giroux, *supra* note 23; Tim Dickinson, *How Republicans Rig the Game*, ROLLING STONE, Nov. 21, 2013, at 36.

[64] Pierce, Elliott & Theodoric Meyer, *supra* note 63; Giroux, *supra* note 23; Dickinson, *supra* note 63. *See also* Cynthia Canary & Kent Redfield, *Partisanship, Representation and Redistricting: An Illinois Case Study*, *Paper #38*, THE SIMON REVIEW OF THE PAUL SIMON PUBLIC POLICY INSTITUTE (2014), http://paulsimoninstitute.siu.edu/_common/documents/simon-review/Canary-Redfield%20Redistricting%20Paper%20Final%20Text.pdf.

extreme outcomes, 8 favored the party that controlled the process, and one was under the control of a nonpartisan commission. Indeed, the extreme cases include all states with single-party control mentioned on a redistricting watchlist[65] published in 2011 by the *Washington Post*.

Later in this article, I will develop a second measure of partisan asymmetry. Both that analysis and the analysis presented here would be aided by a means of evaluating both measures using a comparable statistical yardstick. For this purpose it is convenient to use the standard deviation, sigma. The standard deviation can be used as a natural measure of deviations from the average simulation in terms of excess seats, and as a substitute for what fraction of simulations are as extreme as the actual outcome. The difference from the natural outcome can be divided by sigma to define a quantity, Delta. Generally speaking, for a bell-shaped curve, which these simulations approximately follow, a difference of Delta=1 or more in a particular direction occurs in approximately 16% of cases. A difference of Delta = 2 or more occurs in approximately 2.3% of cases. A difference of Delta=3 or more occurs in approximately 0.13% of cases. Thus Delta is a handy and universal reference measure for detecting extreme outcomes.

Table 1 shows states for which the partisan discrepancy was greater than 1 sigma in 2012. For comparison, discrepancies for the same states are shown for 2010 and 2014. Simulation-based values for sigma are given in the columns labeled "SD (sigma)".[66]

[65] Aaron Blake & Chris Cillizza, *The Top 10 States to Watch in Redistricting*, WASH. POST POLITICS BLOG (Mar. 18, 2011) http://www.washingtonpost.com/blogs/the-fix/post/the-top-10-states-to-watch-in-redistricting/2011/03/18/ABju9Ar_blog.html.

[66] These values are approximated reasonably well by the formula sigma = 0.52 * √(s * (N-s) / N), where N is the number of a state's Congressional districts and s is the average number of seats won in that state by either major party in computer simulations. The principal difference from the "all tossups" example is the appearance of a factor of 0.52, which arises from the fact that some districts are competitive, and some are not; this factor fell within a narrow range of 0.50-0.53 between 2008 and 2014.

Five states showed deviations that were greater than one sigma and less than two sigma: Florida, Illinois, Indiana, Maryland, and Virginia. Six more states exceeded the two-sigma criterion: Arizona, Michigan, North Carolina, Ohio, Pennsylvania, and Texas. Of these eleven states, Redmap's efforts toward redistricting targeted[67] Indiana and all four Republican-controlled states with two-sigma discrepancies: Michigan, North Carolina, Ohio, and Pennsylvania[68]. Of the remaining greater-than-two-sigma states, a fifth state, Texas, was redistricted by Republicans but showed a discrepancy favoring Democrats. A sixth state, Arizona, was redistricted by an independent commission and favored Democrats.

	2010						2012						2014				
	Total seats	Democratic vote share	Dem. Seats	Simulated average	SD (sigma)	Difference in SD	Total seats	Democratic vote share	Dem. Seats	Simulated average	SD (sigma)	Difference in SD	Democratic vote share	Dem. Seats	simulated average	SD (sigma)	Difference in SD
Arizona	8	46.74%	3	3.18	0.78	R by 0.2	9	45.60%	5	2.96	0.76	D by 2.7	43.17%	4	2.94	0.75	D by 1.4
Florida	25	42.53%	6	7.38	1.31	R by 1.1	27	50.00%	10	11.73	1.33	R by 1.3	49.02%	10	12.32	2.59	R by 0.9
Illinois	19	54.96%	8	11.15	1.23	R by 2.6	18	55.40%	12	10.04	1.11	D by 1.8	51.42%	10	9.14	1.12	D by 0.8
Indiana	9	42.04%	3	2.53	0.78	D by 0.6	9	45.80%	2	3.02	0.76	R by 1.3	38.90%	2	2.03	0.72	R by 0.04
Maryland	8	63.42%	6	6.13	0.78	R by 0.2	8	65.46%	7	6.11	0.72	D by 1.2	58.14%	7	5.13	0.76	D by 2.4
Michigan	15	47.87%	6	6.35	1.07	R by 0.3	14	52.70%	5	6.97	0.98	R by 2.0	50.88%	5	6.96	0.98	R by 2.0
North Carolina	13	42.47%	4	3.80	0.94	D by 0.2	13	50.90%	4	5.94	0.93	R by 2.1	45.31%	3	4.89	0.92	R by 2.1
Ohio	18	44.75%	5	6.26	1.14	R by 1.1	16	47.90%	4	6.48	1.03	R by 2.4	40.94%	4	4.40	0.98	R by 0.4
Pennsylvania	19	48.41%	7	8.27	1.21	R by 1.1	18	50.70%	5	8.14	1.10	R by 2.9	46.07%	5	7.06	1.09	R by 1.9
Texas	32	37.51%	7	5.69	1.30	D by 1.0	36	39.90%	12	8.68	1.47	D by 2.3	39.35%	11	8.65	1.43	D by 1.6
Virginia	11	44.94%	3	3.87	0.89	R by 1.0	11	49.00%	3	4.56	0.86	R by 1.8	44.84%	3	4.02	0.84	R by 1.2
Wisconsin	8	45.40%	3	2.91	0.77	D by 0.1	8	50.76%	3	3.64	0.73	R by 0.9	47.20%	3	3.36	0.73	R by 0.5

Table 1: Discrepancies Between Simulated and Actual Delegations for the 2010-2014 House Elections. For 2010, 2012, and 2014, one million simulations were done for each state, resampling was done from nationwide House election returns for that year. The "SD (sigma)" column indicates the value of sigma calculated from the simulations. Color text indicates values of Delta (difference between simulation and actual) exceeding 1 times sigma favoring either party. Shading indicates differences exceeding 2 times sigma. Note the persistence of effects in 2014.

California. As a counterexample to the imbalanced states shown above, the example of California is worth mentioning. California was redistricted by an independent commission. In 2012, the California House popular vote was 62% Democratic resulting in 38 out of 53, or 72%,

[67] *See* THE REDISTRICTING MAJORITY PROJECT, http://www.redistrictingmajorityproject.com/ (last visited Aug. 24, 2015).

[68] Pierce, Elliott & Theodoric Meyer, *supra* note 63; Giroux, *supra* note 23; Dickinson, *supra* note 63.

Democratic seats. However, the average simulated delegation was also 72% Democratic.[69] Thus election results in California exactly meet the expectations that arise from nationwide districting patterns.

Texas. Although the resampling simulations are a powerful and sensitive measure, the case of Texas demonstrates how examination of additional factors can be necessary. Before the 2012 election in Texas, a complex series of legal battles culminated in a court-ordered redistricting plan[70] and a Congressional election outcome in which over 60% of Texas voters voted for Republicans to elect 24 out of 36 seats. From a statistical standpoint, this was an underperformance for Republicans, who in a simulation would have won over 28 seats on average – a discrepancy of Delta = 2.3 times sigma. One major factor contributing to this discrepancy was the presence of Hispanic majorities in seven districts[71], six of which elected Democratic Congressmen. These majority-minority districts, which have special status under the Voting Rights Act of 1965, reflect the growing Hispanic population in Texas, which as of the 2010 Census constituted 32% of Texans[72]. Since Democrats won approximately 40% of the statewide two-party popular vote, wins by Democrats in 12 out of 36 seats (33% of seats)

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[69] A theoretical symmetric distribution of districts would, on average, give a delegation that is 79% Democratic. For a symmetrically distributed distribution of districts whose two-party vote share has standard deviation *SD*, the expected fraction of seats S for a given vote share V is normcdf((V-0.5)/*SD*), where normcdf is the integral of a bell-shaped normal curve with mean 0 and width parameter 1. For non-dysproportional states in 2012, SD=0.15, comparable to longstanding findings for seats-votes curves. GRAHAM GUDGIN & PETER J. TAYLOR, SEATS, VOTES, AND THE SPATIAL ORGANISATION OF ELECTIONS (1979).

[70] *Redistricting in Texas*, BALLOTPEDIA, http://ballotpedia.org/wiki/index.php/Redistricting_in_Texas (last visited Aug. 24, 2015).

[71] *List of Majority Minority United States Congressional Districts*, WIKIPEDIA, https://en.wikipedia.org/wiki/List_of_majority_minority_United_States_congressional_districts (last visited Aug. 24, 2015).

[72] Sharon R. Ennis, Merarys Rios-Vargas & Nora G. Albert, *The Hispanic Population: 2010*, 2010 CENSUS BRIEFS (2011), http://www.census.gov/prod/cen2010/briefs/c2010br-04.pdf

indicate an outcome that is eu-proportional compared with national standards. Thus the final outcome in Texas in 2012 favored the partisan minority relative to nationwide districting patterns, and would not necessarily be grounds for further action.

Florida. In this case, where the value of Delta is between one and two, a similar but statistically stronger answer is given by a map-drawing approach. Chen and Rodden took a geographically intensive approach, drawing districts using automated rules of contiguity and community-preservation, and implemented these rules thousands of times through detailed computer simulation[73]. They found that Florida's 2010 redistricting scheme was more favorable to Republicans than over 99% of their simulations, indicating that the Florida Legislature applied an approach that led to a more partisan outcome than Chen and Rodden's rules would support. Florida's state Constitution mandates specific principles of districting and allows for judicial review by the state Supreme Court[74]. In July 2015[75], the Florida Supreme Court returned the map to the Legislature with instructions to re-draw districts to comply with the state Constitution.

Repairing the one-sigma and greater Republican-redistricted states (seven in all) would lead to an average swing of approximately 28 seats (an average of 27.7) toward Democrats; repairing the two Democrat-redistricted states, Illinois and Maryland, would lead to an average swing of approximately 6 seats (an average of 5.7) toward Republicans. Therefore, based on these measures, Republican gains in 2012 from aggressive redistricting were nearly five times the advantage gained by Democrats from the same process. This sharp asymmetry coincides with

[73] Chen & Rodden, *Unintentional Gerrymandering*, *supra* note 38; Chen & Rodden, *Cutting Through the Thicket*, *supra* note 38; Chen & Rodden, Report on Computer Simulations, *supra* note 38; Chen & Rodden, Supplemental Report, *supra* note 38.

[74] Fla. Const. art. III, §§ 20-21.

[75] *League of Women Voters of Florida v. Detzner*, ---So. 2d---, 2015 WL 4130852 (No. SC14–1905, July 9, 2015) (Fla. 2015).

a period during which state legislative processes have come increasingly under single-party control[76]. Changes between decadal redistrictings favored Republicans, who controlled 13 state capitals in 2002, rising to 24 state capitals in 2012. During that same interval, Democrats went from controlling 8 state capitals to controlling 13 state capitals. Thus the potential for partisan control of districting has increased for both major parties, with a greater advantage for the Republican Party.

4. What accounted for the antimajoritarian outcome of 2012?

With these analytical tools in hand, it is now possible to calculate the total effect of asymmetric partisan districting on the national House elections of 2012. The outcome was a 33-seat margin of control, with 234 Republican and 201 Democratic seats. Applying party-neutral standards to the seven Republican-controlled states and two Democratic-controlled states would have given an average margin that was 22 seats smaller, or 212 Democrats and 223 Republicans. Because of the uncertainty contained in this analysis (a two-sigma uncertainty of six seats), it is just within the range of possibility that without partisan asymmetry, Democrats might have taken control of the chamber.

Republicans have a second advantage, one that arises from population clustering. A simulation-based approach can be used to quantify the net impact of this phenomenon, in which voters self-sort into communities with shared voting patterns. Such self-affiliation facilitates a packing effect by facilitating the drawing of districts that are heavily tilted toward Democrats[77]. The size of this effect can be estimated by computing what share of seats would be expected if

[76] Klarner, *supra* note 24.

[77] Chen & Rodden, *Unintentional Gerrymandering*, *supra* note 38.

district-by-district vote shares were perfectly symmetrically distributed. States in which I did not find dysproportionality had a two-party vote share of 50.7% for Democrats, and 180 out of 363 seats. Simulation of perfect partisan symmetry[78] predicts that this vote share would lead to Democrats winning 51.8% of seats, or 188 seats. The outcome-versus-prediction difference of 8 seats, scaled proportionally to all 435 seats, amounts to Republicans winning 9 or 10 seats more than they would under perfectly symmetric conditions; in other words, a swing of 18 to 20 seats in the margin between the parties. This effect is smaller than the net effect of partisan dysproportionality, and the relative magnitude of these effects is consistent with previous work[79]. That similarity suggests that deviations from natural seats-votes relationships are driven not by political geography, which varies from state to state, but by political motivations and actors during the legislative process.

In summary, partisan redistricting more than doubled the amount of asymmetry caused by natural patterns of population. Together, gerrymandering and population clustering are more than enough to account for the fact that in 2012, Democrats won the House popular vote but Republicans ended up in control of the chamber.

B. ANALYSIS #2: VOTER CLUSTERING BY INTENTIONAL GERRYMANDERING AND SELF-ASSOCIATION

Analysis #1 established a method for identifying states in which voter preferences lead to representation that is anomalous relative to national norms. These anomalies could be rectified

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[78] *See* GUDGIN & TAYLOR, *supra* note 69.

[79] Stephanopoulos & McGhee, *supra* note 25; John Sides, *Not Gerrymandering, but Districting: More Evidence on How Democrats Won the Popular Vote but Lost the Congress*, The Monkey Cage (Nov. 15, 2012), http://themonkeycage.org/blog/2012/11/15/not-gerrymandering-but-districting-more-evidence-on-how-democrats-won-the-popular-vote-but-lost-the-congress/.

through the ballot box: if election outcomes shift sufficiently, legislators can be voted out, thus bringing outcomes more in line with the popular will. As an example of how this mechanism can fail, the election of 2014 heralded a "wave year" in which Republicans won the national popular vote by 5.9%, in sharp contrast to the Democratic popular vote win of 2012. However, in the 12 states in Table 1, Republicans gained control of only five of 187 seats. This small change indicates that representatives in these states were largely insulated from a large swing in opinion from 2012 to 2014. Considering the strength of partisan gerrymandering in 2012, this small change means that Republicans reaped most of their electoral gains two years earlier than their popular support would have merited.

A principal effect of partisan gerrymandering is to reduce the responsiveness of races across a state. Analysis #2 presents a way to quantify the potential for legislative elections to respond to changes in voter opinion.

1. ABSENCE OF CLOSE LOSSES AS AN INDICATOR OF PARTISAN EFFECT

The fundamental strategy in achieving dysproportional outcomes is to identify voters from the opposing party, then pack them into as few districts as possible. The fruits of this procedure create a characteristic lopsided pattern of election results that can be used to identify when packing is likely to have occurred.

State-level gerrymandering is more elaborate than single-district gerrymandering, and relies on a two-part strategy. First, jam voters likely to favor one's opponents into a few throwaway districts where the other side will win lopsided victories, a strategy known as "packing." Second, draw the remaining, more numerous, districts using boundaries that lead to more numerous, more narrowly-won victories. In this process, the critical requirement is asymmetry: the opposing party's voters must be more tightly packed than one's own voters.[80] The net result is an increased likelihood of unrepresentative outcomes.

Here I will examine lopsided patterns in gerrymandered states, and compare them to nongerrymandered states. This allows comparison of the magnitude of the effects of

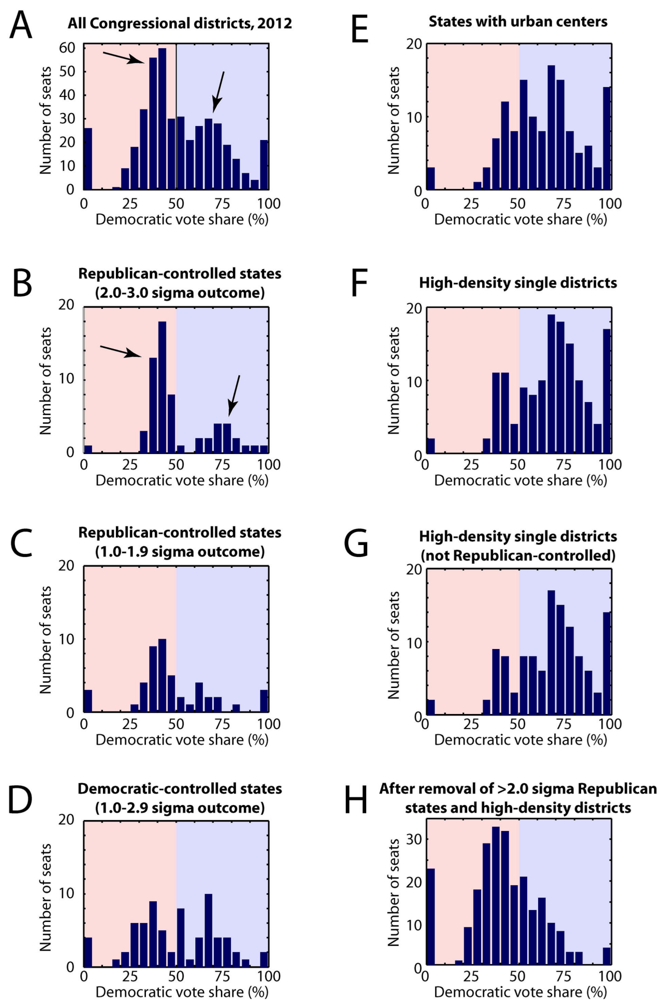

Figure 5: District-by-district histograms of 2012 House election results. (a), All districts, showing one peak with close Republican wins and one peak with lopsided Democratic wins. These peaks persist in **(b)**, districts from states showing an overall advantage to Republicans of 2 times sigma or greater, based on resampling simulations; and **(c)**, districts with a Republican advantage of 1 to 1.9 times sigma. **(d)**, Peaks are shifted or absent in Democratic-advantaged states. **(e)**, States with urbanized populations, defined as greater than 1 million people in cities larger than 250,000, in the 2010 Census. **(f)**, Districts with more than 1000 persons per square mile. **(g)**, Same as (f) but with the districts in (b) and (c) removed. (h), Same as (g) but with the districts in (f) removed.

[80] Because members of both major parties get packed into districts in a partisan gerrymander, individual members of the opposing party may acquiesce or even be complicit in the process. *See*, *e.g.*, *League of United Latin American Citizens v. Perry*, 548 U.S. 399, 418 (2006) (noting "a number of line-drawing requests by Democratic state legislators were honored"). In other words, a single-district gerrymander can favor one party even as a partisan gerrymander favors the other party. For this reason, the use of intent as a standard for gerrymandering should distinguish between district-level and party-level motivations.

gerrymandering and patterns of voter residence and less-partisan districting. This analysis can be used to define Analysis #2, an index of gerrymandering that depends directly on the partisan redistricter's desired goal: the packing of opponents, as measured by election returns.

Gerrymandered districts show a distinctive pattern of lopsided votes (**Figure 5**). Figure 5a shows a histogram of two-party vote share for 2012 House districts that were asymmetric to favor Republicans. In this histogram, two peaks are apparent: a narrow peak centered near a 40% Democratic vote share, and a broader peak centered near a 30% Republican vote share (indicated on the histograms by a 60% to 80% Democratic vote share). Both of these peaks are sufficiently prominent that they can also be seen in a histogram drawn using all states nationwide (**Figure 5a**). The peaks are considerably more prominent when the histogram includes only Republican-favoring states (**Figures 5b and 5c**) or Democratic-favoring states (**Figure 5d**).

However, voter packing can be asymmetric simply by virtue of the fact that voters arrange themselves in a manner that is not symmetric. Therefore any measure of gerrymandering-based packing must be done relative to a baseline of how voters "pack themselves"[81]. Specifically, it has been suggested that structural factors such as concentration of Democrats in urban areas may have a greater effect than partisan redistricting. I will now quantify the size of these two effects. Since both real packing by redistricters and virtual packing by structural factors are likely to have similar manifestations, they can be examined using the same statistical tools.

2. GERRYMANDERING EMULATES THE EFFECTS OF URBANIZATION

[81] Sides, *supra* note 79.

The establishment of competitive districts is often made difficult by the fact that voters often choose to live near others of similar ethnic, religious, secular, and political affiliation. Such self-selection is visible in urban regions that vote overwhelmingly for Democrats, and rural regions that vote overwhelmingly for Republicans. If natural population clustering favors Republicans, then the distribution of vote share in urbanized districts should resemble that of Republican-gerrymandered states. Such a pattern is not apparent in high-population-density states (**Figure 5e**). However, urbanized districts (**Figure 5f**), defined as those with population density greater than 1000 persons/square mile, show both peaks, but with more emphasis on the high-Democratic-vote share peak. This pattern is visible even when putatively gerrymandered states (favoring both Democrats and Republicans) are omitted from the histogram (**Figure 5g**).

Gerrymandering makes use of existing urbanization. In Republican-gerrymandered states, non-urbanized districts (**Figure 5b and 5c**) are dominated by Republican-packed districts, demonstrating that redistricters who seek a Republican advantage do so by creating numerous districts that avoid urban regions. Once Republican gerrymanders and urbanized areas are omitted, the remaining Congressional districts show considerably less tendency to have two peaks (**Figure 5h**).

Although the representational effects of voter migration into urban communities are similar to the effects of partisan gerrymandering, the interpretations of the two phenomena are quite different. Voters who arrange themselves in this manner are voluntarily depriving themselves of the possibility of living in a district where representatives are at risk of losing their seat. In the case of partisan gerrymandering, voters are placed into political affiliation with one another – but without the consent of the citizens involved. Such a pattern contradicts the saying that "voters should choose their representatives, and not the other way around."[82] Gerrymandering thus penalizes voters based on their publicly available information, including partisan loyalty, all of which is present in Census data and commercial redistricting software.

3. THE "CLOSE-LOSS" STANDARD

In summary, the success of a gerrymandering scheme depends on the ability of the redistricting party to lose by large margins in a small number of districts, while winning by narrower margins in the remaining districts. This pattern of outcomes can be quantified by counting how many times the redistricting party suffers close losses, and comparing that frequency[83] to the expectation from national patterns of districting. Based on the histograms in Figure 6, a working definition of "close loss" would be a vote share of 35% to 49.99% for the redistricting party. The proportion of close losses by the redistricting party gives a test that encapsulates Analysis #2.

C. ANALYSIS #3: THE SHIFTED MEDIAN AS A MEASURE OF PARTISAN ASYMMETRY

1. THE MEAN-MEDIAN DIFFERENCE AS A MEASURE OF SKEWNESS

Now that I have identified states in which Republicans or Democrats gained an asymmetric advantage, I can examine these states to test the validity of a simpler statistic that does not require computer simulation: the difference between the mean (*i.e.* average) and the

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[82] Mitchell N. Berman, *Managing Gerrymandering*, 83 TEXAS L. REV. 781 (2005).

[83] Nationwide in 2014, Republicans lost 88 out of 435 races with 35-50% vote share, or one-fifth of all races (20.2% to be exact). Democrats lost 108 out of 435 races with the same vote share, one-fourth of all races (24.8% to be exact). These statistics provide a baseline for predicting what fraction of races within a state would be expected to be similarly close.

median vote share for contested[84] districts. The mean-median difference is a simple measure of asymmetry[85] and allows easy comparison with national standards. Notably, it does not require any inputs other than district-level election results for the state that is under examination.

As an example of the calculation, consider the 2012 Pennsylvania Congressional election. The Democratic two-party share of the total vote in all 18 districts was, in terms of percentages and sorted in ascending order:

34.4, 36, 37.1, 38.3, 40.3, 40.6, 41.5, 42.9, 43.2, 43.4, 45.2, 45.2, 48.3, 60.3, 69.1, 76.9, 84.9, 90.6.

Races won by Republicans are indicated in *italics* and the two middle values are underlined. The median percentage is defined as the midpoint of the two middle values, 43.3%. The mean Democratic vote share is 51.0%. The difference between the median and the mean is 7.7%.

The median serves as a measure of the overall behavior of the 18 district-level elections. The goal of a gerrymander is to maximize the number of districts won, which occurs when the median outcome is more unfavorable to the opposing party than that party's share of the vote. In other words, Pennsylvania's Democratic voters were empowered as if they comprised 43.3% of

[84] The presence of uncontested races reduces the value of the mean-minus-median statistic. In those cases, the partisan breakdown is not known with accuracy. Consider the example of a 20-district state, residents of an uncontested district would have voted at a rate of 80% for their party, instead of the nominal 100%. If their district were drawn differently, the appropriate mean for comparison would be based on the 80% figure, and shift the overall mean by 1%.

[85] The mean-median difference is a simple and old measure of "skewness," a statistical term for asymmetry. David P. Doane & Lori E. Seward, *Measuring Skewness: A Forgotten Statistic?*, 19 JOURNAL OF STATISTICS EDUCATION (2011), www.amstat.org/publications/jse/v19n2/doane.pdf; Karl Pearson, *Contributions to the Mathematical Theory of Evolution, II: Skew Variation in Homogeneous Material*, TRANSACTIONS OF THE ROYAL PHILOSOPHICAL SOCIETY, SERIES A, 186, 343-414 (1895). G. UDNY YULE AND MAURICE G. KENDALL, AN INTRODUCTION TO THE THEORY OF STATISTICS 162-3 (3d ed. 1950).

voters, even though they actually comprised 51.0%. The difference, 7.7%, is the number of voters who were effectively disenfranchised. Since approximately 5,400,000 Pennsylvanians cast votes in the 2012 Congressional election, redistricting achieved an effect equivalent to over 400,000 Democratic voters casting their ballots for Republicans. The probability is less than 1% that this difference arose by chance.

2. State-by-state comparisons of skewness with population clustering effects

To investigate the degree to which the mean-median difference arises as a function of population clustering patterns, I will make comparisons to a variety of states and years. For the 2012 Congressional elections, the nationwide mean-median difference was 4.3% across all 50 states and 1.9% for non-dysproportional states, in both cases favoring Republicans. For Pennsylvania 2012 the difference was 7.7%, greater than any of the other numbers, and comparable to the other four dysproportional states of Michigan (mean-median difference of 6.3%), North Carolina (7.3%), Ohio[86] (6.3%), and Virginia (6.3%). Generally, mean-median differences of greater than 6.0% are reliably associated with dysproportional outcomes and single-party control of redistricting. Overall, these mean-median differences are three to four times that seen in non-dysproportional states, indicating that the effects of partisan gerrymandering are three or four times more intense than the effects of population clustering in cities. Thus redistricting in a handful of states can generate a greater deviation from symmetry than population clustering in all 50 states combined.

[86] In Ohio, one race, the 11th District, was uncontested and won by a Democrat, Marcia Fudge.

III. THREE QUANTITATIVE TESTS OF THE EFFECTS OF PARTISAN GERRYMANDERING

A. A THREE-PRONG STANDARD

Based on the statistical evidence presented so far, I now offer a three-prong test for identifying whether state-level partisan gerrymandering has occurred.

Test 1 (the excess seats test): Calculate whether the outcome of the first election after redistricting was dysproportional relative to a simulated seats-votes curve, and that favors the redistricting party. For a state containing N districts, calculate the difference between the actual seats and the simulated expected number, then divide by σ_1 to obtain Delta[87].

Test 2 (the lopsided outcomes test): Count the number of districts in a state where it lost with 35-49.99% of the vote. Is this number below the expected number from either the national average, or the historical average for that state? Defining L as the national or historical average fraction of closely-lost districts, the expected number of such races in a state consisting of N districts is N*L. Divide the difference between actual close losses and expected close losses by σ_2, defined as $\sqrt{(N*L*(1-L))}$, to get a difference measured in units of sigma.

Test 3 (the shifted median test): In a set of districts of equal population, calculate the difference between the mean and the median vote share. Does this share favor the party that controlled redistricting? For this test, divide the mean-median difference by σ_3 is defined as 0.756 * (standard deviation of vote share across all N Congressional districts in a state)/$\sqrt{N}$.[88]

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[87] This σ_1 can be calculated according to the formula for sigma in note 65.

[88] Paul Cabilio & Joe Masaro, *A Simple Test of Symmetry About an Unknown Median*, 24 THE CANADIAN JOURNAL OF STATISTICS / LA REVUE CANADIENNE DE STATISTIQUE 349 (1996); Tian Zheng & Joseph L. Gastwirth, *On Bootstrap Tests of Symmetry About an Unknown Median*, 8 JOURNAL OF DATA SCIENCE 397 (2010).

Any one test may be susceptible to a false-positive result, i.e. identifying that a gerrymandering event occurred when in fact it did not. As a conservative test to minimize the risk of a false-positive determination, I propose that conditions for partisan gerrymandering can be defined as a combination of all three tests giving at least a 1-sigma difference, in all cases favoring the same party. In 2012, five states met these criteria: Michigan, North Carolina, Ohio, Pennsylvania, and Virginia.[89]

	Test #1 (simulation)			Test #2 (lopsided losses)				Test #3 (skewed districts)					
								directly from election returns			imputing uncontested races		
	Total seats	Simulated average	Δ (difference divided by sigma)	Close losses by redistricting party	Expected	sigma	Fewer than national average by sigma?	Average minus median Dem vote (%)	sigma	Δ (Average-median divided by sigma)	Average minus median Dem vote (%)	sigma	Δ (Average-median divided by sigma)
Arizona	9	2.96	D by 2.7	2	2.2	1.3	no	-0.5%	3.8%	D by 0.1	-3.3%	3.8%	D by 0.9
Florida	27	11.73	R by 1.3	5	5.5	2.1	no	4.8%	3.8%	R by 1.2	4.8%	2.4%	R by 2.0
Illinois	18	10.04	D by 1.8	4	4.5	1.8	no	2.1%	3.1%	R by 0.7			
Indiana	9	3.02	R by 1.3	2	1.8	1.2	no	1.4%	2.1%	R by 0.7			
Maryland	8	6.11	D by 1.2	0	2.0	1.2	yes (D)	-2.8%	3.9%	D by 0.7			
Michigan	14	6.97	R by 2.0	1	2.8	1.5	yes (R)	6.9%	3.7%	R by 1.9			
North Carolina	13	5.94	R by 2.1	0	2.6	1.4	yes (R)	7.8%	3.2%	R by 2.5			
Ohio	16	6.48	R by 2.4	1	3.2	1.6	yes (R)	6.8%	4.3%	R by 1.6	6.8%	3.0%	R by 2.3
Pennsylvania	18	8.14	R by 2.9	1	3.6	1.7	yes (R)	7.6%	3.2%	R by 2.4			
Texas	36	8.68	D by 2.3	3	7.3	2.4	yes (R)	4.9%	3.1%	R by 1.6	7.0%	2.4%	R by 2.9
Virginia	11	4.56	R by 1.8	1	2.2	1.3	yes (R)	6.3%	3.4%	R by 1.9			

Table 2: Results of the Three-Prong Test for Partisan Asymmetry for The Congressional Elections of 2012. In all cases, the last column gives the difference between expectations and actual result, expressed in units of sigma, the standard deviation, to give a measure that is comparable across the three tests. Test #3 is done starting from raw percentage results, and also taking uncontested races and assuming that their voters are distributed 75%-25% for the winning party.

[89] A threshold of 1 sigma has the advantage not only of statistical simplicity, but also that in the case of Test 1, in most cases it corresponds approximately to a discrepancy of 1 Congressional seat. If all three tests are independent, the occurrence of meeting all three tests with a 1-sigma threshold would arise by chance in only 16%*16%*16%, or 0.4% of cases, a high degree of statistical uncertainty. It should be noted that the three tests arise from the same dataset, they may not be perfectly independent of one another. In a worst-case scenario, if the three measures are totally non-independent, they are equivalent to measuring Delta just once. In this case, a value of Delta=1 would correspond to an approximately 16% chance that the outcome arose by nonpartisan means. To safeguard against the possibility of non-independence, an additional standard could be set that at least one test should give a value of Delta that exceeds 2. In this case, the probability of a false-positive outcome is reduced to less than 5%. This additional condition would – just barely – lead to the omission of Virginia from the list. Another possible adjustment is to omit two-sigma states from the population of districts to be sampled in constructing fantasy delegations, on the grounds that they are presumptively gerrymandered. In this case, the Test-1 deviation for Virginia increases to 2.0 sigma, sufficient to meet the higher threshold for significance.

B. ADVANTAGES OF THE THREE-PRONG STANDARD

The three-prong effects standard has five advantages. First, the tests are simple to apply and do not require the detailed drawing of maps. Second, these tests are directly related to goals of representative democracy. Test 1 addresses whether a redistricting scheme leads to an elected delegation that more proportionally represents the wishes of voters: deviations from national districting norms should either remain neutral or move closer toward proportional representation. This principle resembles the "one man, one vote" standard, but is expressed in terms of obtaining legislative representation for men/women and votes. Test 2 uses the number of closely-fought districts to test whether a party in control of redistricting has eliminated the possibility of electoral competition. Test 3 measures a parameter that partisan gerrymanderers seek to shift, the median partisan vote. Third, these criteria do not require evaluation of intent. Finally, because of the simplicity of these tests, they can easily be combined to reduce the risk of falsely identifying a gerrymander where none occurred. Finally, the three-prong test does not use geography and can easily be augmented and complemented by state-level districting standards.[90]

C. TWO EXAMPLES: THE ORIGINAL GERRY-MANDER AND ARIZONA STATE LEGISLATIVE DISTRICTS.

To examine the general applicability of these tests, let us consider two examples: (1) the original Gerry-mander of 1812, and (2) post-2010 Arizona state legislative districts (*Harris v.*

[90] The three-prong effects standard addresses the overall apportionment plan, but does not cover the case of individual self-dealing. Local laws may provide additional constraints. For example, the current Congressional districts in Florida do not violate the three-prong standard. Nonetheless, the Florida Supreme Court has found the map to violate the Florida Constitution redistricting provisions (article III, section 20(a) that reads "No apportionment plan or district shall be drawn with the intent to favor or disfavor a political party or an incumbent"). *Detzner*, 2015 WL 4130852. This stricter standard extends a mandate for competitive races to the level of single districts.

Arizona Independent Redistricting Commission, to be considered by the Supreme Court in its 2015-16 session).

Example 1: The original "Gerry-mander," the Massachusetts state Senate election of 1812. Test 1 is evaluated by starting from the fact that there were 18 races.[91] The average expectation of a nearly evenly divided popular vote is 9 races for each party. The upper theoretical value to sigma is $0.5*\sqrt{18} = 2.1$ races; computational simulation reveals a true value of sigma of 1.4 races. The Federalists won only five races, and therefore Test 1 is met to a standard of $(9-5)/1.4 = 2.9$ sigma.

For Test 2, the Democratic-Republicans lost five races (which accounted for 11 districts); in these races, their two-party vote share was between 23% and 33%; in other words, zero races fell within the 35-50% vote-share range. For comparison, in 1811, three of their 15 (i.e. 3/15=0.2) of races were in the same range. The value of sigma is $\sqrt{18*0.2*(1-0.2)} = 1.7$ races. Since 3 is greater than 1.7, Test 2 is satisfied.

Test 3 cannot be calculated because the condition of equal-sized districts is violated. In 1812 the number of votes per legislator ranged from Dukes/Nantucket (1,078 votes cast in total for 1 legislator) to Franklin (4,469 votes for 1 legislator).

Example 2: Arizona state legislative districts. After the 2010 Census, the Arizona Independent Redistricting Commission, which is composed of members of both major political parties, drew House and state legislative districts. Currently before the Supreme Court for the 2015-16 term, *Harris v. Arizona Independent Redistricting Commission* concerns "[W]hether the desire to gain partisan advantage for one political party justifies intentionally creating over-populated legislative districts that results in tens of thousands of individual voters being denied

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91 In that election, multimember districts of unequal population were allowed. For the calculation of Test 1, each district election is used as one data value.

Equal Protection because their individual votes are devalued, violating the one-person, one-vote principle."[92] In this case plaintiffs contended, and the District Court of Arizona "assume[d], without deciding, that partisanship is not a valid justification for departing from perfect population equality."[93] Defendants contended that the construction of districts of unequal population was done in order to comply with Section 5 of the Voting Rights Act.

Although the issue at hand is the creation of overpopulated districts, neither side has contested in federal courts the premise that the Commission created a partisan advantage. Nonetheless, the question bears examination: did redistricting actually create a partisan advantage in the first place? This question can be tested by examining State Senate races, of which there is one for each of Arizona's 30 legislative districts.

Test 1 relies on computer simulation using other comparable districts as a source of hypothetical districts. The statewide two-party popular vote totaled 56.3% for Republicans and 43.7% for Democrats, yielding 17 seats for Republicans and 13 seats for Democrats. Because other states have different districting systems (for instance with different numbers of people per district), data is not available for the type of seats-votes of calculation outlined previously. However, a similar calculation is possible: proportional representation would predict 16.9 seats for Republicans. Therefore the election result is almost perfectly euproportional.

For Test 2, plaintiffs have asserted that the Democratic Party benefited. In 2010, 11 out of 30 Senate seats were lost by Democrats with a vote share in the 35-49.99% range. For a probability L of 11/30=0.37, the calculated sigma is 2.6 seats. In 2012, 9 out of 30 Senate seats

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[92] *Case File for Harris v. Arizona Independent Redistricting Commission*, SCOTUSBLOG, http://www.scotusblog.com/case-files/cases/harris-v-arizona-independent-redistricting-commission/ (last visited Aug. 24, 2015).

[93] *Harris v. Arizona Independent Redistricting Commission*, 993 F.Supp.2d 1042 (D. Ariz. 2014) at 42.

met this condition, a decrease of 2 seats in a direction favoring Democrats. However, because 2 is less than 2.6, the difference does not meet the 1-sigma standard.

For Test 3, the mean Democratic vote share across 30 districts was 50.1%, and the median was 45.6%. The difference is a gap of 3.3% (4.1% with imputation) in a direction that favors Republicans. This difference works against Democrats, and therefore is in the wrong direction.

Based on the foregoing, Arizona Senate districts fail all three tests. Therefore the contention that Democrats benefited in a dysproportional manner is not supported.

IV. DISCUSSION

The statistical analysis of state-specific partisan asymmetry in this article shows that in 2012, the effects of partisan gerrymandering were so large as to exceed the effects of population clustering across the whole nation. Both natural and redistricting-driven segregation generate seats that are unresponsive to shifts in public opinion, in the first case because voters have clustered themselves geographically, in the second case because legislators have corralled them using the redistricting pen. The health of democratic processes would be considerably improved by reducing the artificial, legislatively-driven component. The three-prong test for asymmetry provides a means of identifying when this artificial clustering has occurred.

A. IS THE INTENT STANDARD STILL NEEDED?

These tests are independent of, and indeed do not address, the question of intent. The initial requirement of an intent prong in *Bandemer* presented a higher bar to proving injury than evaluation of disparate impact alone. This may have been appropriate in the absence of legislative guidance or court precedent. In the *Bandemer/Vieth* framework, the lack of simple and reliable tests made it necessary to assess the link between redistricters' actions and the injury. Indeed, current approaches to proving gerrymanders focus on intent, are diverse in approach, and sometimes do not agree with one another[94].

An example of ambiguous intent is found in *LULAC v. Perry*. The Republican majority was able to involve individual Democratic legislators in the districting process. However, in matters of redistricting, a party as a whole has motivations that can be at odds with those of some of their own party's individual legislators[95]. Therefore intent is most fairly evaluated at the state level or at the individual level, but not both at the same time. In addition, the majority in Crawford *et al. v.* Marion County Election Board *et al.* held that partisan intent is insufficient as a reason to strike down voting restrictions[96].

I suggest that districting can impose a burden on a group's representational rights whether or not the effects are intentional. Even where intentions are nonpartisan, bipartisan, or unknown, the effect of a districting plan with partisan asymmetry is to produce legislative blocs whose size is unrepresentative of the popular will. The construction of a reliable and normative measure for effect provides clear guidance when an injury has taken place, and a template for how the injury can be repaired. Just as a road worker may act to right an upended orange traffic cone even if he/she does not know how the cone came to be tipped over, a court may act when

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[94] Micah Altman, Brian Amos, Michael P. McDonald, & Daniel A. Smith, *Revealing preferences: why gerrymanders are hard to prove, and what to do about it*, SOCIAL SCIENCE RESEARCH NETWORK, March 22, 2015. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2583528. Last accessed August 27, 2015.

[95] *supra* note 80.

[96] *Crawford v. Marion County. Elec. Bd.*, 553 U.S. 181 (2008).

effects are sufficiently strong, as in disparate impact cases in racial discrimination cases[97]. Although partisan gerrymandering cases are governed by different principles (equal protection and/or First Amendment doctrine), the question remains of the manner in which partisan intent is taken into account. At a minimum, it is necessary to separate the questions of statewide and individual-district partisan intent when evaluating the intents standard of the *Bandemer* test.

B. USES AND LIMITS OF THE QUANTITATIVE STANDARD

The three-prong effects-based analysis of the effects of gerrymandering may be of particular relevance to First Amendment analysis, which "allows a pragmatic or functional assessment that accords some latitude to the States."[98] By allowing for a normal amount of statistical variation, the three-prong test proposed in this article builds in zones of ambiguity where any of a range of outcomes would lead to an acceptable amount of asymmetry.

Any statistical approach contains some possibility of accidentally identifying gerrymandering where it does not exist (in statistical terminology, "false positives"), or missing cases where it did occur (false negatives). For this reason I have provided three separate tests that can be applied. These tests are oriented toward the outcomes of elections rather than the specifics of map boundaries or district procedures. Also, because the tests hew closely to the electoral goals of redistricters, they contain fewer assumptions that are present in more intricate, geographically-oriented statistical tests.

[97] In one recent example, in a racial discrimination case the Supreme Court ruled that demonstration of disparate impact was sufficient to prove discrimination, and that a demonstration of intent was not necessary. *Tex. Dep't of Housing and Comm. Aff. v. Inclusive Communities Project,* 136 S. Ct. 2507 (2015).

[98] *See e.g.*, *Eu v. San Francisco County Democratic Central Comm.*, 489 U. S. 214. (1989).

In current federal precedent, the need for redrawing a set of districts often relies on forensic evidence; that is, on elections that have already occurred. However, by that time an injury to voters has already occurred. Although in this article the present three-prong standard was applied using election results, it can also use as its inputs information that is available before an election. Under the First Amendment rationale of not penalizing groups for their partisan preference, voter registration might be considered a form of self-identification that can be used as an input to the three-prong standard. Other variables are used to predict partisan preference; such prediction is done both by political scientists and by redistricters themselves. Commercial redistricting packages can be used to estimate partisan vote shares, which could serve as inputs to the test. Doing so will require consideration of whether courts should evaluate a districting scheme before an election based on the scheme has taken place.

The standard presented here can quantify the benefits of reform efforts directed at reducing the likelihood of partisan gerrymandering. One such route is the establishment of nonpartisan districting commissions that remove districting from the direct control of legislators. In California[99], a voter referendum in 2008 established the formation of the California Citizens Redistricting Commission. The commission is composed of 14 members who are drawn from members of the general public, including five Democrats, five Republicans, and four members who decline to state a partisan loyalty. The commission's mandate is to draw districts that respect principles of contiguity, compactness, and representation of a community's interests. The resulting Congressional districts have become more competitive: margins of victory have become smaller, and incumbents have lost their re-election races at higher rates than before the formation of the commission. Like the Arizona commission, the work of the California

[99] CALIFORNIA CITIZENS REDISTRICTING COMMISSION, http://wedrawthelines.ca.gov/regulation_archive.html (last visited Aug. 24, 2015).

commission has led to closer races[100] and more euproportional overall outcomes than pre-commission maps.

These tests could also be used in approaches that leave districting under the control of state legislators, but place constraints on the districting process. Such an approach has been taken in Florida, ballot initiatives known as Amendments 5 and 6 were passed in 2010, becoming Article III, §§ 20 and 21 of the Florida Constitution[101]. Together with Article III, § 16[102], the Florida Constitution stipulates that district lines "must be contiguous, compact, and use existing political geographical boundaries where available." Districts also may not be drawn to "favor or disfavor a political party or incumbent."[103] The resulting plans are automatically submitted to the Florida Supreme Court for review, leading either to approval or return to the legislature for a further attempt to meet districting criteria. My tests could be useful in identifying statewide partisan favor. Individual districts would still need to be evaluated separately.

At last, if legislation-based approaches to minimizing partisan districting fail, a final approach is judicial intervention. In this case, the three-prong standard may prove to be manageable as a means of determining the effects of partisan gerrymandering.

[100] Quantitatively, an increase in competitive races can be described quantitatively as making the distribution of two-party vote share (dS/dV) curve unimodal instead of bimodal. Making districts more responsive to changes in voter sentiment is equivalent to maximizing dS/dV in a region where V most frequently occurs.

[101] Justin Levitt, *Florida,* ALL ABOUT REDISTRICTING, http://redistricting.lls.edu/states-FL.php (last visited Aug. 24, 2015).

[102] Fla. Const. art. III, § 16.

[103] Fla. Const. art. III, §§ 20-21.