A nearly linear time algorithm for testing membership in minor closed families of graphs

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A model of a graph $H$ in a graph $G$ consists of a set of disjoint trees indexed by the vertices of $G$ such that if $uv$ is an edge of $H$ then there is an edge of $G$ joining $T_u$ and $T_v$. We say that $H$ is a minor of $G$ if $G$ contains a model of $H$. Many classes of graphs are closed under the taking of minors. For example $G$ is planar precisely if it has neither $K_{3,3}$ nor $K_5$ as a minor. For fixed $H$, Robertson and Seymour gave an $O(|V(G)|^3)$ algorithm to determine if $G$ has $H$ as a minor, and if so to find a model of $H$ in $G$.

We present a nearly linear time algorithm for the same problem. Our algorithm also constructs a well-behaved decomposition of an input $G$ which has no $H$-minor which can be used to solve a variety of optimization problems on such a graph.

This talk avoids technical details as much as possible. Much of it is a gentle introduction to some of the ideas involved in this area.