How our work is quoted in [DHRT]

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Abstract

We draw attention to the inadequate way in which our works [KS18], [GCM1], [GCM2] are quoted in the more recent work [DHRT][1].

1 Introduction

Both [KS18] and [DHRT] address the issue of the nonlinear stability of the Schwarzschild spacetime. Since it is known that a general perturbation of a Schwarzschild metric may converge to a slowly rotating Kerr, both make restrictions on the initial data to ensure that the final state is Schwarzschild. In [KS18] it is assumed that the initial data is polarized[2] while [DHRT] deals with a more general codimension 3 set of initial conditions. In [KS18], in addition to ensuring that the final state remains Schwarzschild, polarization simplifies the construction of GCM spheres[3]. A generalized version of a GCM sphere was later constructed in [GCM1], [GCM2] for general perturbations of Kerr and applied to an unconditional proof of nonlinear stability[4] of slowly rotating Kerr.

[1] We posted [KS18] on arXiv in November 2017 and completed a revised version of it in December 2018. We posted [GCM1], [GCM2] on arXiv respectively in November and December 2019. [DHRT] was posted on arXiv in April 2021.

[2] That is axially symmetric plus an additional condition ensuring that the ADM angular momentum vanishes.

[3] GCM (spheres) stands for generally covariant modulated spheres. They play a fundamental role in our construction of GCM admissible spacetimes in [KS18] and, as we point out in this note, a similar construction appears in [DHRT], albeit with no reference to our works. [GCM1], [GCM2] do not even appear in the bibliography of [DHRT].

[4] The paper [KS:Kerr] appeared on arXiv a week after [DHRT]. For a short introduction to the full proof of the nonlinear stability of slowly rotating Kerr, with historical references and bibliography, see [KS:Kerr-Brief].

The main concerns raised in this note are as follows:

- The paper [DHRT] relies crucially on constructions which are very similar to those made in [KS18], [GCM1], [GCM2]. Yet the similarities are not addressed at all in [DHRT]. In particular no references are made to [GCM1], [GCM2].
- The authors of [DHRT] refer instead to their linear paper [DHR] as the main source of these constructions. Yet, as we show in section 3, the constructions in the linear context[5] of [DHR] are not at all those that are implemented[6] in [DHRT].

As of now, after more than 18 months, no corrections have been made. We are left with no other options than to either complain about our concerns in private, or to make them public. We feel that this second option is the more ethically correct one.

In section 2 we describe the main similarities between [KS18], [GCM1], [GCM2] and [DHRT], which are not acknowledged in [DHRT]. In section 3, we review the constructions in the linear context of [DHR] and compare them with those that are implemented in [DHRT]. In section 4 we describe the extent to which our work is misquoted in the introduction to [DHRT]. Section 5 spells out the main things we have asked the authors of [DHRT] to have corrected. Finally, we believe that the way our work is quoted in [DHRT] raises serious questions concerning the AMS guidelines "on mathematical research and its presentation"; we copy these guidelines in section 6 and let the reader decide.

[5] As well known, nonlinear stability results rest, ultimately, on some very specific version of a linearized theory in which precise choices are made and robust analytic methods are implemented in order to derive estimates which can be closed through a bootstrap argument. By contrast in linear theory one has the freedom to stop whenever something interesting has been proved; there is no need for optimality since nothing needs to be *closed*. Thus [DHR] can call their result a "linear stability of Schwarzschild", even though the decay rates derived in the paper are less than what is needed in a proof of nonlinear stability. Another case in point is the nonlinear stability of the Minkowski space where linearized theory may be understood to involve nothing more than the standard flat wave equation. By contrast the nonlinear result in [CK93] required a host of new constructions and methods as well as a whole book to implement them. A more realistic linear theory is treated in [CK90], though even that falls far short of what is needed in the full nonlinear setting.

[6] This can be easily checked, without going into all the details of section 3, by observing that the decay estimates obtained in that paper (See (250)–(252) and (254) in [DHR]) are clearly sub-optimal and thus inapplicable to the nonlinear theory in [DHRT]. For this reason, [DHRT] uses a different gauge, which turns out to be closely related to the one made in [KS18] and [GCM2].

2 Analogies without reference to our work

The following elements of [DHRT] are either in [KS18], or in the GCM papers [GCM1] [GCM2], but not in [DHR]:

1. Analogies in the setup:

   (a) As in [KS18], the bootstrap region consists of two spacetime regions, each region being covered by different foliations[7]. In [DHR], there was only one space-time region, i.e. the complement of the black hole region of Schwarzschild in the future of the initial data set, covered by a double null foliation.

   (b) As in [KS18], the most important feature of the bootstrap spacetime in [DHRT] is its last sphere S_* that satisfies GCM conditions[8]. Not only are these GCM conditions almost identical to the ones of [KS18], albeit without polarized symmetry, but they are in fact the restriction to the particular case $a = 0$ of the general GCM conditions[9] of our second GCM paper [GCM2], which was posted on arXiv 18 months before [DHRT] (up to a minor exchange between the $\ell = 1$ mode of $\breve{\mu}$ and $\widetilde{\mathrm{tr}\underline{\chi}}$).

   (c) As in [KS18][10], the values u_f and $\underline{u}_\infty$ are such that $u_f \ll \underline{u}_\infty$, see (5.6.3) on page 114 of [DHRT].

   (d) As in the second GCM paper [GCM2][11], the angular momentum[12] is computed from the $\ell = 1$ modes of curl β on the GCM sphere S_*, see (V.10.2) on page 27 and Definition 2.3.4 page 53 in [DHRT].

2. Analogies in the proof:

   (a) As in [KS18], the null structure equations in ${}^{(ext)}\mathcal{M}$ are integrated backwards in r, starting from the last slice (analogous to "$\mathcal{I}_+$" in linear theory). In [DHR], all integrations were being performed forward in r from the future horizon $\mathcal{H}_+$.

[7] In [KS18], these two regions are called ${}^{(ext)}\mathcal{M}$ and ${}^{(int)}\mathcal{M}$ (separated by a timelike boundary $r = r_0$), while the corresponding gauges are called respectively $\mathcal{I}_+$-gauge and $\mathcal{H}_+$-gauge in [DHRT].

[8] In [DHRT], the last sphere is called $S(u_f, v_\infty)$. In particular, the GCM conditions on $S(u_f, v_\infty)$ are given by the third to sixth equation in Definition 2.2.1 of [DHRT].

[9] For our GCM conditions in perturbations of Kerr, see (7.6) (7.7) in Theorem 7.3 of [GCM2].

[10] See the identity (3.3.4) in [KS18] relating u_* and r_*.

[11] See (7.16) (7.17) in [GCM2] for our definition of the angular momentum in perturbations of Kerr.

[12] The similarity is particularly striking, since this definition, at the level of first derivatives of the curvature, does not appear very natural at all, unless one takes into account the specific constraints posed by the nonlinear theory. In fact, in the linear context of [DHR], the definition is given in terms of the $\ell = 1$ modes of the σ component of the curvature, see Theorem 9.2 in [DHR]. See [Rizzi] for an even more natural choice, based on the $\ell = 1$ modes of ζ.

(b) As in [KS18], there is a specific separate treatment of $\ell = 0$ and $\ell = 1$ modes, see the general principle in section 14.3.2 of page 301 in [DHRT]. In [DHR], the $\ell = 0$ and $\ell = 1$ modes are identically 0.

(c) As in [KS18], in the context of a bootstrap argument, there are intermediary steps in [DHRT] concerning the control of the initial data layer, the existence of a non trivial bootstrap spacetime, and the extension of the bootstrap spacetime (respectively Theorem M0, Theorem M6 and Theorem M7 in [KS18]). None of these steps exist in the linear context of [DHR].

Unfortunately, none of the above striking similarities between [DHRT] and our earlier works [KS18], [GCM1], [GCM2] are acknowledged, and no references to our papers are given in that context. In fact, as already mentioned, the last two papers are not even mentioned in the bibliography.

3 Comments on [DHR]

Here are some comments on [DHR] concerning their treatment of the linear stability of Schwarzschild:

1. There are two distinct gauges in Chapter 9 of [DHR], the initial gauge of Theorem 9.1 of [DHR] in which boundedness is proved[13], and the final gauge of Proposition 9.3.1 of [DHR] in which decay is proved. Both gauges are constructed by adding a suitable member of the kernel to force the suitable linearized gauge. What looks the closest to the construction of a GCM sphere[14] are the conditions (193) and (194) on the vanishing of the linearized $\Omega \mathrm{tr}\chi$ and linearized mass aspect function. These conditions are enforced using Proposition 9.2.2. Note however that there are never conditions on three linearized scalars on a single Schwarzschild sphere[15]: indeed, in Proposition 9.2.2, conditions (193) and (194) are enforced on the sphere at the intersection of initial data with the future horizon $\mathcal{H}_+$, but the third condition, i.e. (191),

[13] Note that decay does not hold in the initial data gauge of Theorem 9.1 of [DHR]. In contrast, in the nonlinear case, one cannot prove boundedness in a gauge without being also able to prove decay in the same gauge (for a lower number of derivatives).

[14] Note that in the simplified context of linear theory, "constructing a sphere" amounts to solve linearized equations on a fixed sphere of Schwarzschild.

[15] At the nonlinear level, a suitable choice of three scalars leads to the construction of a codimension 2 sphere, see Chapter 9 in [KS18] for Schwarzschild in polarized symmetry and [GCM1], [GCM2] for general perturbations of Kerr. The construction of a codimension 2 sphere was one of the main innovations in [KS18].

is the round sphere condition at infinity, i.e on a different (asymptotic) sphere. This means in particular that there is no "sphere" (i.e. linearized sphere) construction[16] in [DHR], but rather something that would correspond, in the nonlinear setting, to the construction a whole initial data hypersurface in the initial data layer. It is not at all clear whether such a construction could be implemented in a nonlinear setting.

2. Nothing is required on the last sphere of the spacetime in the final gauge[17] which is stated in Proposition 9.3.1 of [DHR]. In fact, the only gauge which is required to hold, anywhere else than on the initial data, is a gauge condition along $\mathcal{H}_+$ for the linearized lapse, see (212), which is easily achieved using the ODE (in (214) of [DHR]) along the horizon. Also, this ODE is initialized from initial data, see again (214) in [DHR]: it is in fact analogous to constructing a new foliation[18] on $\mathcal{H}_+$ from initial data, reminiscent of the construction of the canonical foliation[19] along the last null slice in [KN03]. Thus, in the end, all the gauge fixing in [DHR] is done from initial data, and hence nothing is "teleological" in that paper.

3. The definition of the angular momentum in Theorem 9.2 of [DHR] is not on the final sphere of the spacetime (as in [GCM2] and [DHRT]), but in fact on the sphere at the intersection of initial data with the future horizon. Also, the definition in [DHR] is through the $\ell = 1$ mode of the linearized σ, see Theorem 9.2 of [DHR] (rather than that of curl β as in [GCM2] and [DHRT]).

4. In [DHR], the $\ell = 0$ and $\ell = 1$ modes are identically 0 once the linearized Kerr solution has been subtracted, see the proof of Theorem 9.2 in [DHR].

5. In [DHR], there are no intermediary steps concerning the control of the initial data layer, the existence of a non trivial bootstrap spacetime, and the extension of the bootstrap spacetime (respectively Theorem M0, Theorem M6 and Theorem M7 in [KS18]).

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[16] Note also that in Proposition 9.2.3, there are three conditions, but only for the $\ell = 1$ modes, so that it does not correspond to a sphere construction at the linearized level either. Indeed, constructing a sphere a the linearized level would require to fix the $\ell \geq 2$ modes as well.

[17] The initial gauge is in Definition 9.1. The final gauge in Proposition 9.3.1 has only one extra condition for the linearized lapse, (212), which holds on $\mathcal{H}_+$. (213) is only a condition on the added gauge $\widehat{\mathcal{G}}$. In particular, the final gauge is the initial gauge together with (212).

[18] Indeed, the added gauge is constructed using Lemma 6.1.1 which is consistant with a refoliation of $\mathcal{H}_+$ from initial data, i.e. a change of speed $\tilde{v} = v + \epsilon f_2(v, \theta, \phi)$, see (158), along fixed geodesics foliating $\mathcal{H}_+$ from initial data.

[19] The canonical foliation in [KN03] also has an ODE on the speed with a RHS (depending on the old geodesic foliation)- ODE which is propagated from initial data. The first place where such a construction appears is in [CK93] on a fixed spacelike hypersurface, called last slice, initialized at spacelike infinity.

6. Chapter 6 in [DHR] is about the complete description of the linearized kernel, both induced by general covariance and the linearized mass and angular momentum. This freedom is then used to construct the specific linearized gauges in Chapter 9 of [DHR] for which boundedness and partial, sub-optimal, decay of the linearized perturbations are established.

7. A strong simplification in the linear setting of [DHR] is the fact that quantities that are initially 0 and satisfy a nice transport equation stay exactly zero. This is in particular the case of $\ell = 0$ and $\ell = 1$ modes (see the last paragraph of Chapter 9 of [DHR]), but also of some quantities along the horizon $\mathcal{H}_+$, see Proposition 9.4.1 of [DHR]: indeed, in both cases, the corresponding quantities are assumed to vanish only at initial data, and are then automatically 0 also in the future.

Summary:

1. *"Teleological gauges".* In our opinion, a "teleological gauge" should be one which is not connected to the initial data layer through transport equations[20]. In that sense, there is nothing teleological in [DHR]. Indeed, the only potential "teleological gauge", called the "future normalized gauge" in [DHR], is in fact initialized from the initial data layer and then transported along $\mathcal{H}_+$. This concerns the linearized lapse, whose construction, at the linearized level, is reminiscent to those of [CK93] and [KN03] in the full nonlinear setting.

2. *GCM conditions:* In linear theory, to construct a "sphere" would amount to prescribe three linearized scalars on the same Schwarzschild sphere. As such, there are no "spheres" constructed in [DHR]. It appears however that [DHR] constructs a "linearized" codimension-1 hypersurface in the initial layer. It is highly unlikely that such a construction could be implemented in a nonlinear setting, and in fact the authors of [DHRT] abandon such a construction in favor of the one used in [KS18] and [GCM1], [GCM2]. The reason for that is clear: the gauge fixing from initial data in [DHR] leads to *sub-optimal decay estimates* for some of the metric coefficients[21] and is thus inapplicable to the nonlinear case[22].

3. *Angular momentum.* In [DHR], the angular momentum is defined from the $\ell = 1$ modes of σ, unlike [DHRT] where the angular momentum is defined, as in [GCM2], from the ones of curl β.

[20]The first time such gauges appeared is in [KS18]. The novelty there, to repeat, was the construction of a "far away" codimension 2 sphere not connected to the initial data.

[21]See (250)–(252) and (254) in [DHR].

[22]This deficiency was fixed in the PhD thesis of E. Giorgi in the more general context of the linear stability of Reissner-Nordström see [Giorgi], by relying, precisely, on a linearized version of the GCM construction in [KS18].

4 How is our work quoted in [DHRT]

Our paper paper [KS18] is quoted as follows in [DHRT]:

1. Quotation on page 3: *"This is the statement of Corollary III.3.1. Thus in particular, our submanifold contains also the polarised axisymmetric data considered recently in [KS18], which itself is an infinite codimension subfamily of general axisymmetric data."*

   This statement is problematic in various ways. To start with, the main result in [DHRT] requires a lot more decay on the initial data[23]. It is claimed that these initial conditions can be relaxed in the axially symmetric case, but this is not actually done in [DHRT], and no precise statement is provided to back it up. Moreover the authors of [DHRT] seem to suggest that somehow the polarization assumption makes [KS18] a simple corollary of a corollary of their work ignoring the fact that:

   (a) By comparison to [DHRT], polarization only appears to simplify the construction of the GCM spheres, fact later removed[24] in [GCM1], [GCM2].

   (b) [DHRT] relies in an essential way on constructions which appear first in [KS18], [GCM1], [GCM2] as discussed above.

   (c) The technical difficulties in [DHRT] are totally comparable with those of [KS18], paper which has appeared on arXiv two and a half years earlier than [DHRT].

2. Quotation on page 16: *"Like the present work, the starting point of [KS18] is the linear theory [DHR] discussed in Section II, though [KS18] opts for a different gauge which is still however governed by transport equations of geometric quantities associated to foliations."*

   As acknowledged[25] in [KS18], and repeated in [KS:Kerr] as well as in all our talks on the subject, the treatment of the Teukolsky equation in [DHR], based on a physical space version of the Chandrasekhar transformation, played an important role in our work. But the way the statement is phrased in the quote above suggests that the whole paper [KS18] is based on [DHR], which is of course false, as can be seen from the discussion in sections 2 and 3.

[23] The fact that [KS18] requires less decay than [DHRT] is nowhere to be found in the introduction of [DHRT]. This is important in view of Christodoulou's well known result on the lack of smoothness of scri in [Chr02].

[24] In fact [GCM1], [GCM2] do not only remove polarization, they also apply to general perturbations of Kerr in the full sub-extremal case.

[25] See the introduction to that paper as well as the acknowledgment section.

3. Quotation on page 16: *"Several of the non-linear difficulties discussed above already arise in [KS18] in simplified form and are addressed in that work in a different framework."*

   Apart from the simplification in the construction of GCM spheres (induced by polarization), which was removed in our GCM papers [GCM1], [GCM2], and the actual selection of the co-dimension 3-set of initial data in [DHRT], the difficulties in [KS18] are completely comparable with those of [DHRT]. Once the GCM sphere S_* is constructed, polarization plays a minimal role in [KS18].

Remark 4.1. *There is no acknowledgement that [KS18] has influenced any part of [DHRT]. Also, as mentioned before, there is no reference at all to the two GCM papers [GCM1] [GCM2], posted on arXiv 18 months before [DHRT].*

5 Corrections Proposal (November 8 2021)

This section contains our proposal for changes to the introduction of [DHRT] discussed with I. Rodnianski in November 2021.

The following aspects of the introduction to [DHRT] are problematic to us and we hope that they could be corrected:

1. **Choice of S_*.** With regard to the sphere S_{u_f,v_∞} anchoring the $\mathcal{I}_+$ gauge in [DHRT], the following similarities with [KS18] and [GCM2] should be emphasized:

   (a) Your conditions on S_* are the same as those for the GCM spheres in [KS18] and [GCM2].

   (b) Unlike [DHR], the construction of your gauge starts from S_*, exactly as in [KS18].

   (c) The co-dimension 2 sphere S_* is in the future boundary of the bootstrap spacetime, and not connected to the initial data layer through transport equations. Such a construction has first appeared in [KS18], and is conceptually different[26] from that of [CK93] (or [DHR]).

2. **Definition of angular momentum.** The fact that the angular momentum is defined on S_* from the $\ell = 1$ modes of curl β as in [GCM2], and unlike [DHR].

[26] It is also the only condition that we know of which might be called "teleological", precisely because it is not connected to the initial data layer through transport equations.

3. **Current quotations of [KS18] in [DHRT]:**

   (a) Quotation on page 3: *"This is the statement of Corollary III.3.1. Thus in particular, our submanifold contains also the polarised axisymmetric data considered recently in [KS18], which itself is an infinite codimension subfamily of general axisymmetric data."*

   This statement is problematic in various ways. To start with, you need a lot more decay on the initial data[27]. You claim that these initial conditions can be relaxed in the axially symmetric case, but this is not actually done in your paper, and no precise statement is provided to back it up. But even if it was true, it does not mean it is the right way to present it. Announcing on page 3 that [KS18] is a corollary of a corollary of your result seems unnecessary, and mean spirited[28], especially since this is repeated a second time on page 16 after the corollary. For the sake of fairness we hope that you may either remove that statement, or provide substantive evidence for the claim.

   (b) Quotation on page 16: *"Like the present work, the starting point of [KS18] is the linear theory [DHR] discussed in Section II, though [KS18] opts for a different gauge which is still however governed by transport equations of geometric quantities associated to foliations."*

   As acknowledged in [KS18], and repeated in [KS:Kerr] as well as in all our talks on the subject, the treatment of the Teukolsky equation in [DHR], based on a physical space version of the Chandrasekhar transformation, played an important role in the "gestation of our own ideas". But the way the statement is phrased in the quote above may suggest that the whole paper [KS18] is based on [DHR], which is of course false. At the very least, you should be more specific and point out that [KS18] uses the treatment of the Teukolsky equation in [DHR]. If you think that other parts of [KS18] were influenced by [DHR], you should make precise statements, and avoid what may look like an insinuation that our work is derivative.

   (c) Quotation on page 16: *"Several of the non-linear difficulties discussed above already arise in [KS18] in simplified form and are addressed in that work in a different framework."*

   Apart from the simplification in the construction of GCM spheres (induced by polarization), dispensed with in our GCM papers, and the actual selection of the co-dimension 3-set in your work, the difficulties in [KS18] are no less severe

[27] The fact that [KS18] requires less decay than [DHRT] is nowhere to be found in the introduction of [DHRT]. This is important in view of Christodoulou's well known result on the lack of smoothness of scri in [Chr02].

[28] The same statement is an obnoxious (to us) presence in all talks by Mihalis on the subject.

than in [DHRT]. Once the GCM sphere S_* is constructed[29], polarization plays a minimal role in [KS18]. Could you please be more specific about the way in which these "difficulties are simplified"?

4. **Other quotations:**

   (a) The following quotations:

   - the last sentence of the abstract: *"In view of the recent [DHR19, TdCSR20], our approach can be applied to the full non-linear asymptotic stability of the subextremal Kerr family"*;
   - the following sentence on page 3: *"With these more recent developments, the whole approach of this work can in principle be generalised to Kerr, in fact in the full subextremal range of parameters"*;
   - and the following sentence on page 19: *"In view of our discussion and the recent [DHR19, TdCSR20], the path is now open to obtaining Conjecture IV.1 following the approach of the present work, although, at a technical level, the Kerr case introduces several new complications related to the necessity of applying frequency localisation to deal with the issues related to trapping."*

   Your paper provides no substance to that repeated claim, which may thus be interpreted as a claim of priority for any future advance in the field. We hope that you either remove the claim or provide solid evidence to it. At this stage, apart from our own works [GCM1] [GCM2], which are completely ignored in [DHRT], and [KS:Kerr], we are not aware of any work, that gives even a glimpse on how to construct the correct gauges in perturbations of Kerr.

   (b) The following sentence on page 12: *"With these results, the full linear stability of Kerr in analogy to [DHR] can in principle be obtained, in fact, for the entire subextremal range $|a| < M$."*

   The only evidence you give for that claim is the recent work by Yacov and Rita on the control of Teukolsky in Kerr. There are of course plenty of other issues to solve, including the issue of gauges, before being able to make such a claim. We hope you may also reconsider the statement in light of the fact that our GCM papers [GCM1] [GCM2] are not restricted to small angular momentum so that, in a sense they solve the gauge problem in the full subextremal case.

We are also seriously concerned about repeated statements such as *"[KS18] is based on [DHR]"*, or *"[KS18] is a corollary of a corollary of [DHRT]"*, or that *"[DHRT] provides a*

[29] Which you keep saying that it is not a big deal anyway. If that is indeed the case, why ignore the GCM papers [GCM1] [GCM2], which are done in full generality in perturbations of Kerr?

roadmap for Kerr". These have appeared not only in print in [DHRT], but are repeated in all talks on the subject by Mihalis, as well as talks and papers of his students. The failures to properly quote our papers, as well as all these unsubstantiated claims, have done great damage to the visibility and reputation our own works, damage which we hope can be repaired, at least in part, by taking the actions which we recommend above.

6 AMS guidelines on mathematical research

We reproduce below the AMS guidelines on mathematical research and its presentation, see https://www.ams.org/about-us/governance/policy-statements/sec-ethics.

The public reputation for honesty and integrity of the mathematical community and of the Society is its collective treasure and its publication record is its legacy. The knowing presentation of another person's mathematical discovery as one's own constitutes plagiarism and is a serious violation of professional ethics. Plagiarism may occur for any type of work, whether written or oral and whether published or not. The correct attribution of mathematical results is essential, both because it encourages creativity, by benefiting the creator whose career may depend on the recognition of the work and because it informs the community of when, where, and sometimes how original ideas entered into the chain of mathematical thought. To that end, mathematicians have certain responsibilities, which include the following:

- To endeavor to be knowledgeable in their field, especially about work related to their research;
- To give appropriate credit, even to unpublished materials, materials on websites, and announced results (because the knowledge that something is true or false is valuable, however it is obtained);
- To publish full details of results that are announced without unreasonable delay, because claiming a result in advance of its having been achieved with reasonable certainty injures the community by restraining those working toward the same goal;
- To use no language that suppresses or improperly detracts from the work of others;
- To correct in a timely way or to withdraw work that is erroneous.

A claim of independence may not be based on ignorance of widely disseminated results. On appropriate occasions, it may be desirable to offer or accept joint authorship when independent researchers find that they have produced identical results. All the authors listed for a paper, however, must have made a significant contribution to its content, and all who have made such a contribution must be offered the opportunity to be listed as an author. Because the free exchange of ideas necessary to promote research is possible only when every individual's contribution is properly recognized, the Society will not knowingly publish anything that violates this principle.

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