Myths, Facts and Dreams in General Relativity

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6. A good concept of **local** or **quasi-local mass** is needed, to understand how trapped surfaces form.
7. Black holes cannot form **in vacuum**, i.e. absence of matter.

1. Are Existence and Uniqueness Results Superfluous ?

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  - Introduction of non-physical energy norms
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  - Functional analytic setting for proving existence
- Provides a framework for the main conjectures in GR.

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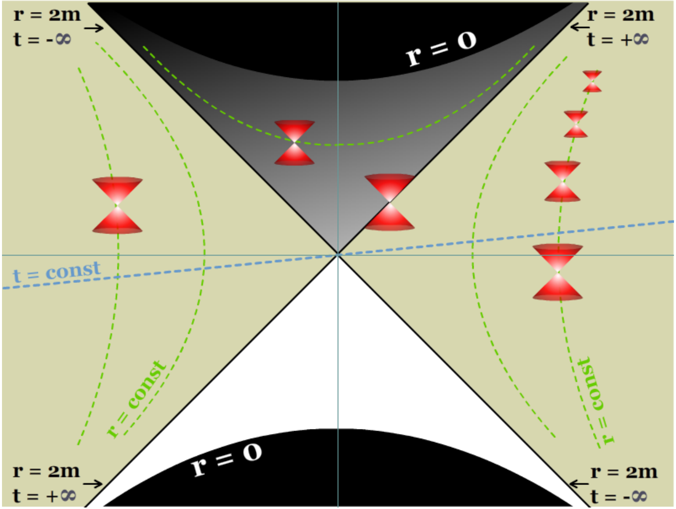
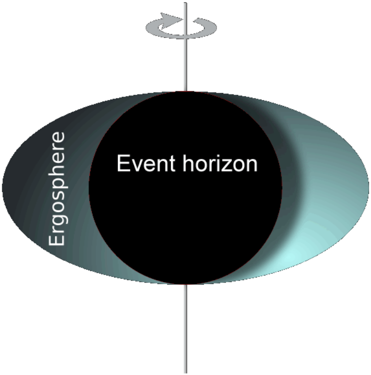
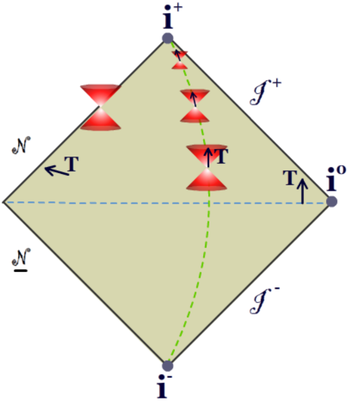
Conjecture(FSC) MFGHD of complete, **asymptotically flat**, **generic**, initial data sets have maximal future developments which look, asymptotically, in any finite region of space, as a member of the Kerr family $\mathcal{K}(a, m)$, $0 \leq a < m$.

$$-(1-\frac{2m}{r})dt^2+(1-\frac{2m}{r})^{-1}dr^2+r^2d\sigma^2_{\mathbb{S}^2}$$



Rotating Black Holes; Kerr Solutions $\mathcal{K}(a, m)$





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Conjecture[BCC.] The Bruhat-Geroch theorem holds true for initial data with bounded curvature in L^2 of the initial hypersurface.

Breakdown Criteria

Theorem [Kl-Rodnianski, Wang] Any spacetime $(\mathcal{M}, g)$ endowed with a maximal, space-like foliation Σ_t can be smoothly continued, beyond $t = t_*$ as long as the second fundamental form k and lapse n of the foliation verify the scale invariant condition

$$\int_0^{t_*} \left(\|k(t)\|_{L^\infty(\Sigma_t)} + \|\nabla n(t)\|_{L^\infty(\Sigma_t)} \right) dt < \infty$$

- Require uniform bounds for the curvature tensor R using a geometric parametrix formula.
- To be operative the parametrix requires a uniform lower bound for the radius of injectivity of null backward light cones.

2. Positive Mass and Stability of Minkowski space.

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Fact: Despite misleading statements to the contrary the second theorem does not follow from the first. New stability results do not even require a finite ADM mass !

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Proof gives a rigorous definitions of null infinity, Bondi mass, news function, Penrose diagram. **Laws of gravitational radiation**.

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Fact. Analyticity is not at all a reasonable assumption.

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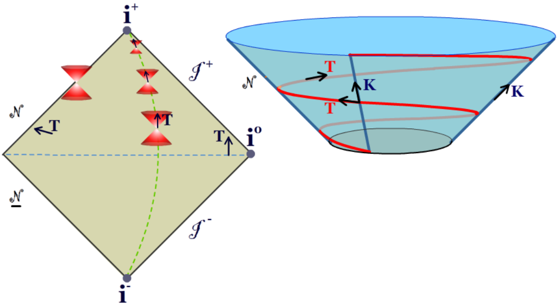

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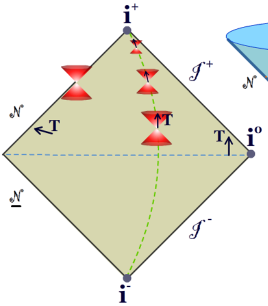

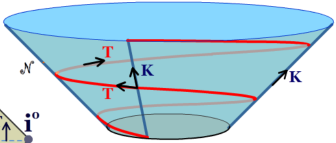

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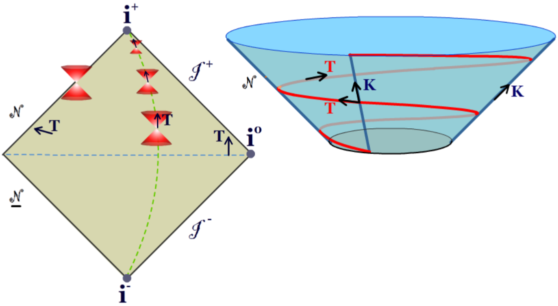

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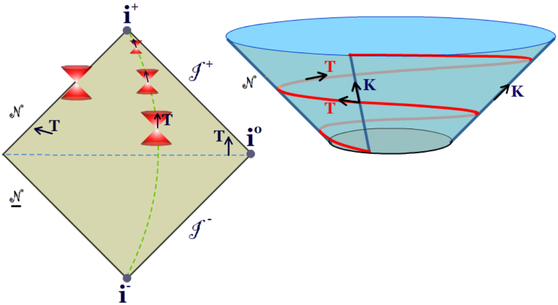

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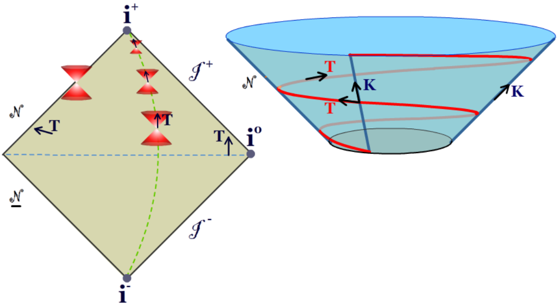

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5. Deduce axi-symetry and apply Carter-Robinson

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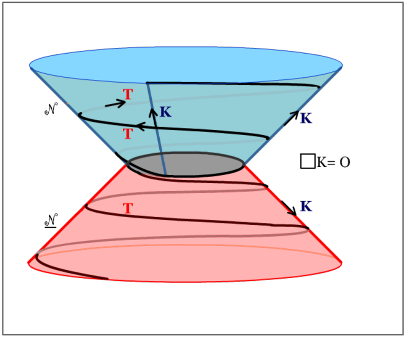

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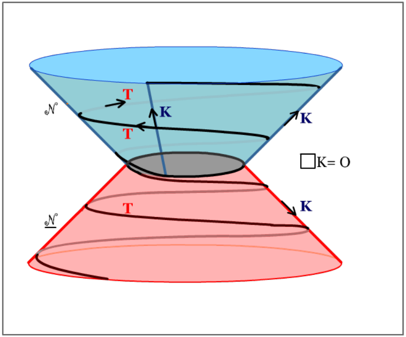

Fact. Bifurcate horizon (i.e. **nondegenerate**) is essential. Are degenerate stationary black holes unique ?

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Is there a non-linear version of the Holmgren's uniqueness theorem?

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1. trapped null geodesics,
2. super-radiance.
3. degeneracy of the horizon.

Fact. These difficulties have been, **recently**, overcome by mathematicians, using a substantial **extension** of the vector-field method. (Soffer-Blue, Blue-Sterbenz, Dafermos- Rodnianski, Tataru-Tohaneanu, Blue-Anderson).

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- Patching of non-causal vectorfields

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Hope. Such a quantity must exist for general asymptotically flat space-times.

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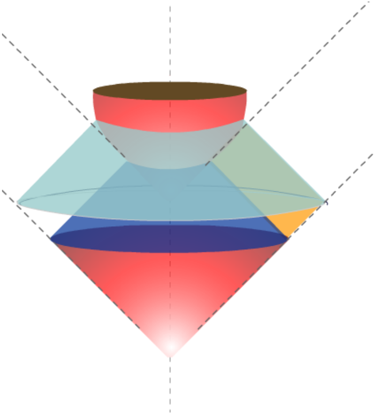
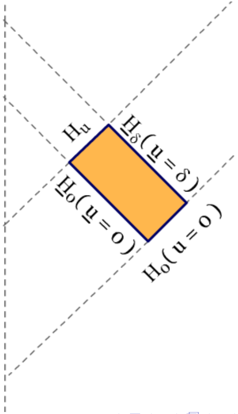
Theorem[Chr. 2008] Specify regular, **characteristic**, initial data, in **vacuum**, and show that its future development must contain a trapped surface





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Kl-Rodnianski (2010) Introduce a different scaling, allowing localizations in angular sectors, which vastly simplifies the proof while providing a stronger result. Lower bound is only uniform in **most** directions.