Hodge correlators, Hodge symmetries, and Rankin-Selberg integrals

A.B. Goncharov (Yale University)

Abstract

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Rankin-Selberg integrals, among many other things they do, are the only way to prove that special values $L(f, n)$ of L-functions of weight k modular forms on $GL_2(\mathbb{Q})$, $n \geq k$, are periods. They pave the road to Beilinson's motivic ζ-elements, organized by Kato into an Euler system.

We define Hodge correlators – a collection of complex numbers given by certain integrals associated with a complex variety X. When X is a modular curve, the simplest of them literally coincide with Rankin-Selberg integrals, and the entire collection consists of periods of mixed motives which are iterated extensions of the motivic ζ-elements.

The destination of Hodge correlators is to encode an explicit homotopy action of the $\mathbb{R}$-Hodge Galois group by A_∞-equivalences of the derived category of holonomic D-modules on X, providing a glimpse of the coveted Hodge site. When X is a modular curve, this explains the origin of the motivic ζ-elements.

If F is a number field, Rankin-Selberg integrals on $GL_2(F)$ suggest a generalization of Hodge correlators related to the modular manifolds, which are rarely algebraic. I conjecture that, nevertheless, they always deliver periods, while their meaning is completely obscure.