SEMINAR

TITLE: Impossible intersections for elliptic curves.

ABSTRACT: We proved with Umberto Zannier that there are at most finitely many complex numbers $\lambda \neq 0, 1$ such that two points on the Legendre elliptic curve $y^2 = x(x-1)(x-\lambda)$ with coordinates $x = 2$ and $x = 3$ both have finite order. However we still do not know how to find these λ effectively (there are probably none). We discuss various extensions of this result, some of them effective. For example some comments of Serre encouraged us to show that there are no λ at all for coordinates $x = u$ and $x = v$ when the complex numbers u, v are not both algebraic with $1, u, v$ linearly dependent over the field of rationals and $uv(u-1)(v-1)(u-v) \neq 0$. The proof needs a close study of what may be called "bicyclotomic polynomials".