Abstract

Given a family $F$ of $r$-uniform hypergraphs, the classical Turán theory studies the maximum proportion of $r$-sets an $n$ element set can have without containing $F$. This limit, as $n$ becomes large, is sometimes called the Turán density of $F$. We consider the related problem of determining the (normalized) maximum possible minimum co-degree that the family of $r$-sets in an $n$ element set can have without containing $F$. As $n$ goes to infinity, this approaches a limit which we call the co-degree density of $F$, and write $g(F)$. For each $r > 1$, let $G_r = \{g(F) : F$ is a family of $r$-graphs$\}$. Our main result is that for each $r > 2$, $G_r$ is dense in $[0, 1)$. This is in stark contrast to the fact that $G_2 = \{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$\}, a fact that follows from the Erdős-Simonovits-Stone theorem. This phenomenon is similar to the existence of real numbers that are not jumps in hypergraphs, proved by Frankl and Rödl. Several other results about co-degree densities are provided parallel to those of the classical Turán theory.

This is a joint work with Dhruv Mubayi.