Aspects of the multivariate Tutte polynomial (alias Potts model) for graphs and matroids

Abstract

The multivariate Tutte polynomial (known to physicists as the Potts-model partition function) can be defined on an arbitrary finite graph $G$, or more generally on an arbitrary matroid $M$, and encodes much important combinatorial information about the graph (indeed, in the matroid case it encodes the full structure of the matroid). It contains as a special case the familiar two-variable Tutte polynomial — and therefore also its one-variable specializations such as the chromatic polynomial, the flow polynomial and the reliability polynomial — but is considerably more flexible. I begin by giving an introduction to all these problems, stressing the advantages of working with the multivariate version. I then discuss some questions concerning the complex zeros of the multivariate Tutte polynomial, along with their physical interpretations in statistical mechanics (in connection with the Yang–Lee approach to phase transitions) and electrical circuit theory. Along the way I mention numerous open problems. This talk is intended to be understandable to mathematicians with no prior knowledge of physics.