Abstract

A graph \( G \) is said to be \( k \)-edge choosable if for every set system \( S_e : e \in E(G) \) with each \( |S_e| = k \) there exists a proper edge coloring \( c \) such that \( c(e) \in S_e \) for every \( e \in E(G) \). The list edge coloring conjecture states that every \( k \)-edge colorable multigraph is \( k \)-edge choosable. Ellingham and Goddyn verified this conjecture for \( d \)-edge colorable \( d \)-regular planar multigraphs by proving that all \( d \)-edge colorings of such multigraphs have the same sign. Goddyn conjectured that if a \( d \)-edge colorable \( d \)-regular multigraph \( G \) admits a Pfaffian orientation then all of its \( d \)-edge colorings have the same sign.

We prove Goddyn’s conjecture for a slightly larger class of multigraphs that admit a ”Pfaffian labeling”. Conversely, we prove that if a multigraph does not admit a Pfaffian labeling, then by adding parallel edges we can obtain from it a \( d \)-regular multigraph with two \( d \)-edge colorings of different signs. Furthermore, we prove that all the graphs that have Pfaffian labellings can be constructed in a certain way from Pfaffian graphs and the Petersen graph. We also describe graphs with Pfaffian labellings in terms of their drawings in the projective plane.

This is joint work with Robin Thomas.