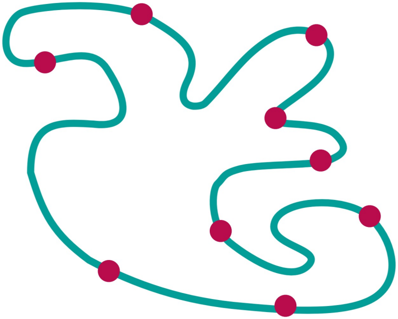
Colouring visibility graphs

Rose McCarty

Joint work with:
James Davies, Tomasz Krawczyk, and Bartosz Walczak

Matroid Union Seminar
October 2020



Theorem

Any visibility graph with clique number ω has chromatic number at most $3 \cdot 4^{\omega-1}$.

- A class is χ-bounded if $\chi \leq f(\omega)$
- Was open for polygon visibility graphs (Kára-Pór-Wood).
- It is natural to consider hereditary graph classes.

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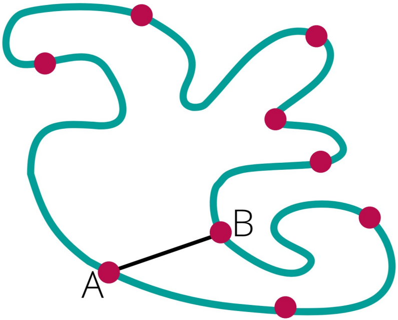

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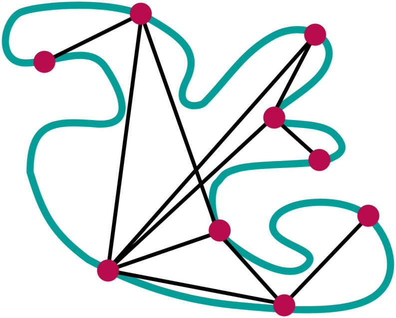

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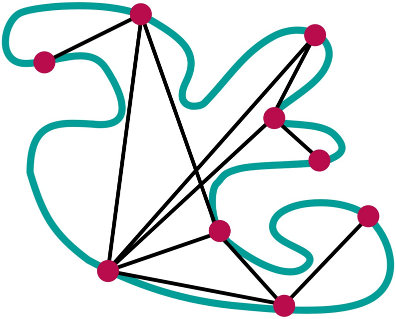

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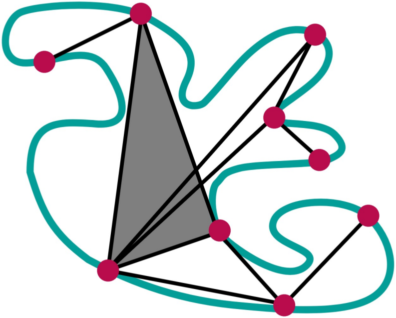

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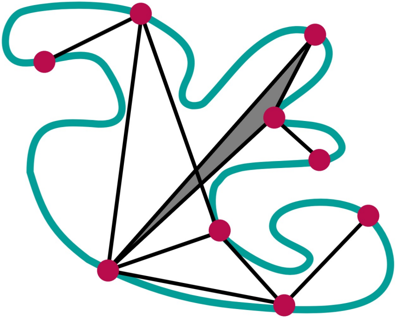

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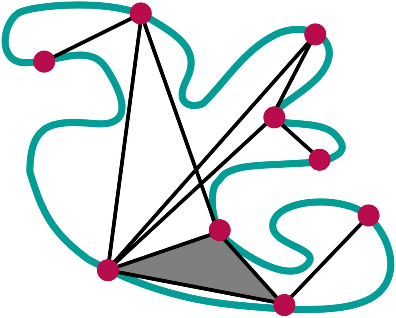

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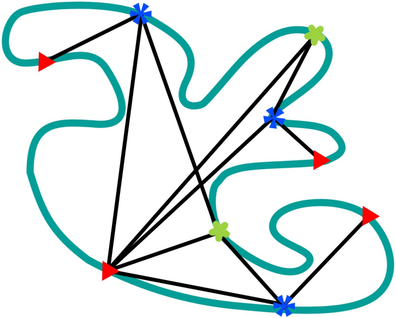

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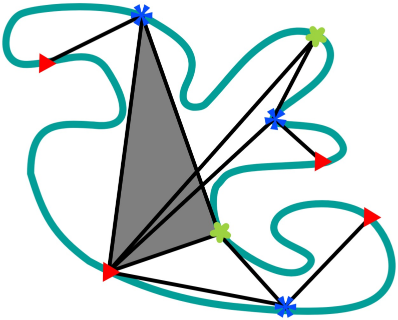

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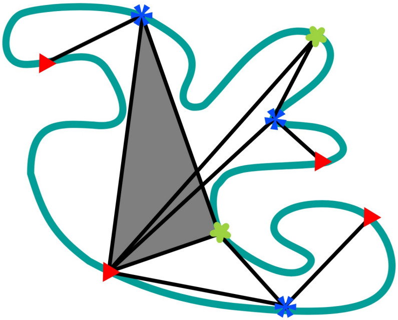

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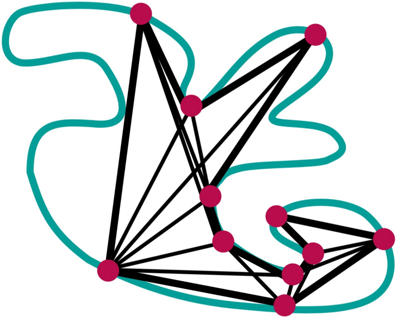

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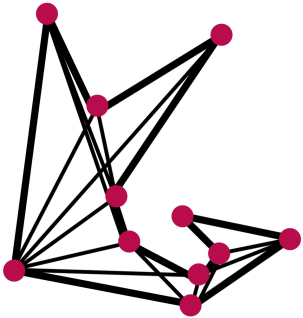

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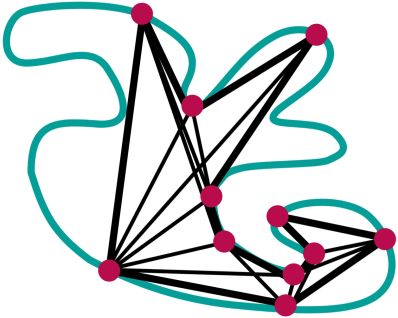

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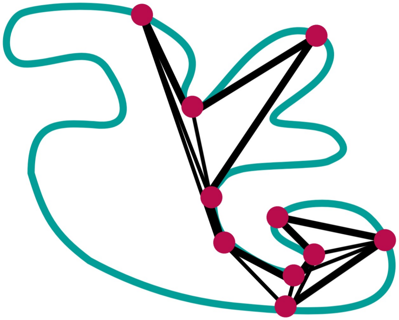

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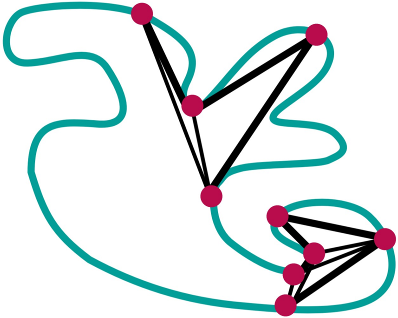

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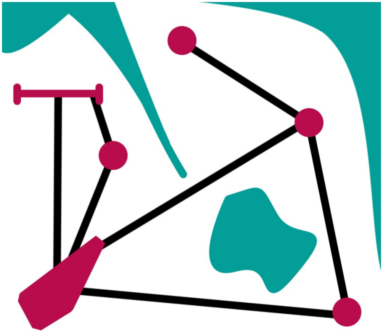

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All intersection/disjointness graphs of curves:

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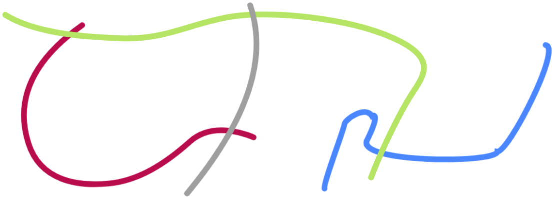

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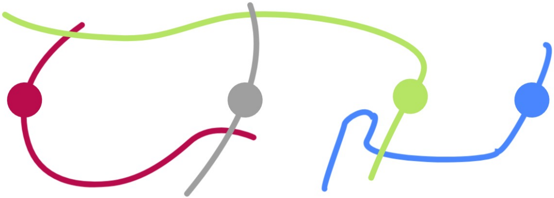

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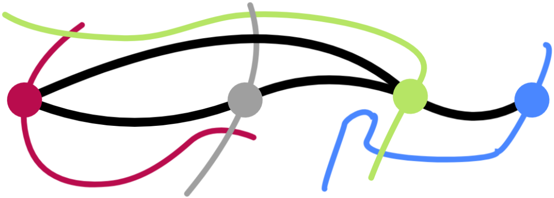

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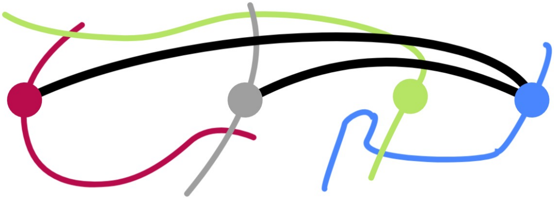

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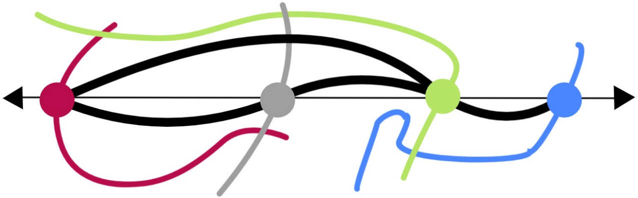

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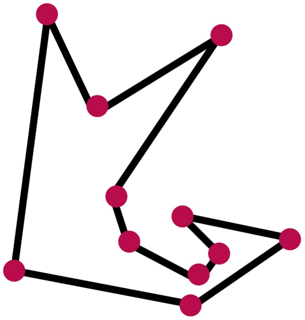
Polygon visibility graphs are fundamental in computational geometry.



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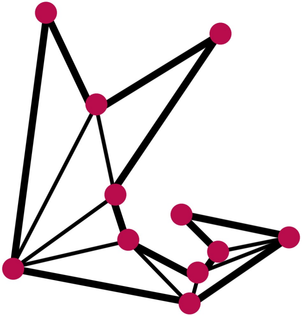

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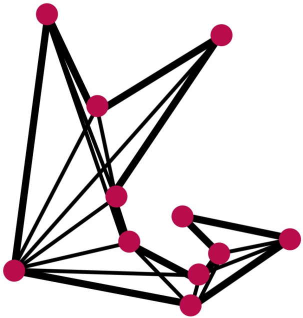

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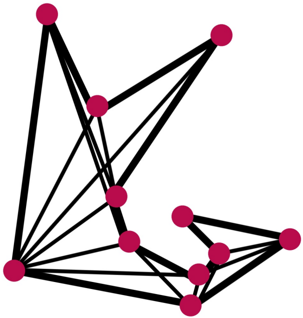

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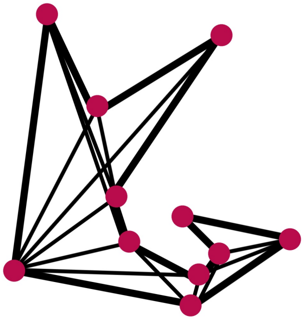

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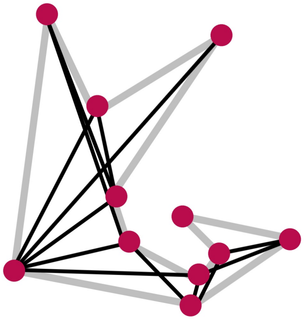

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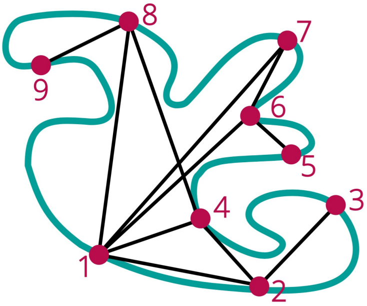

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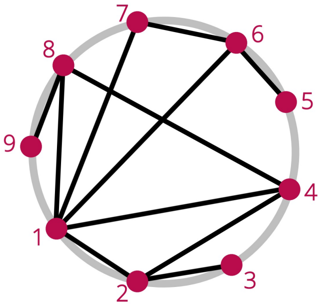

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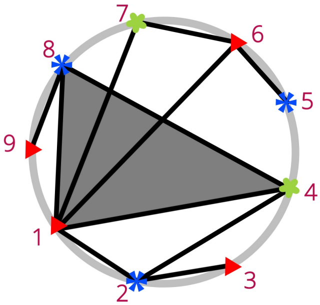

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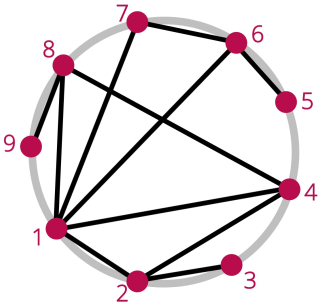

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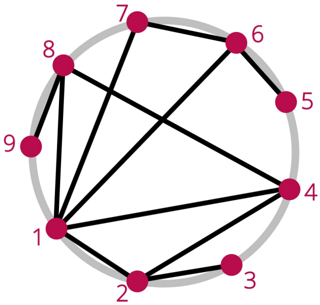
Conjecture

The recognition problem is NP-hard.

Is it in NP? (Ghosh-Goswami 13)



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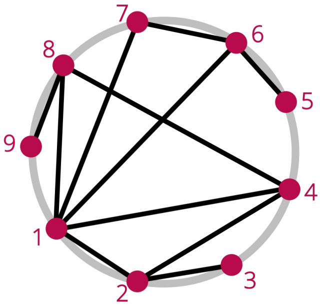
Hamiltonian $\longrightarrow$ in NP (O'Rourke-Streinu 97).



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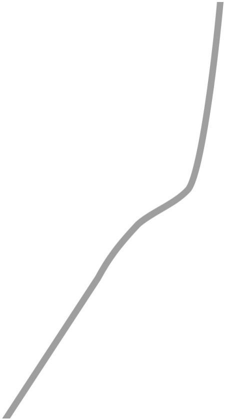
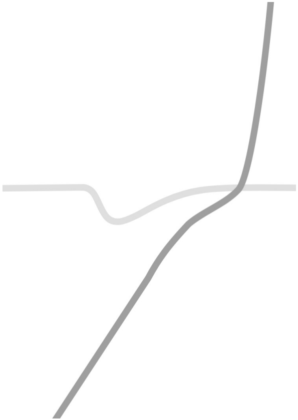


A **pseudoline** in a pseudoline arrangement of size $\binom{4}{2}$, yielding a pseudolinear drawing of K_4 on the unit circle.

This defines a pseudo-visibility graph.



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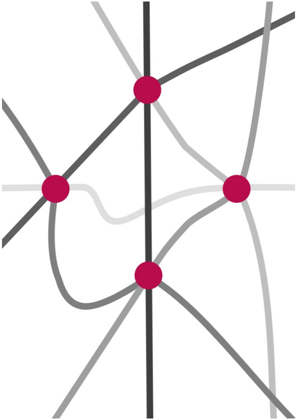

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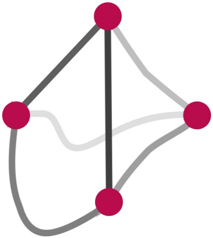

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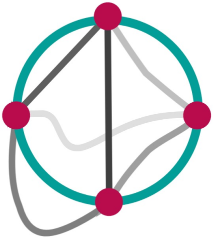

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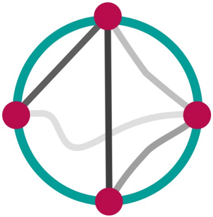

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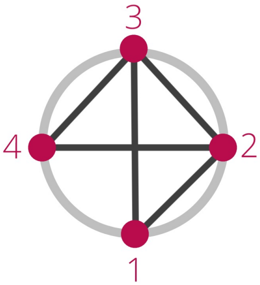

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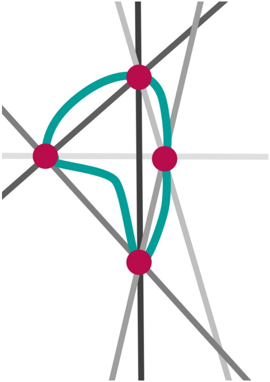
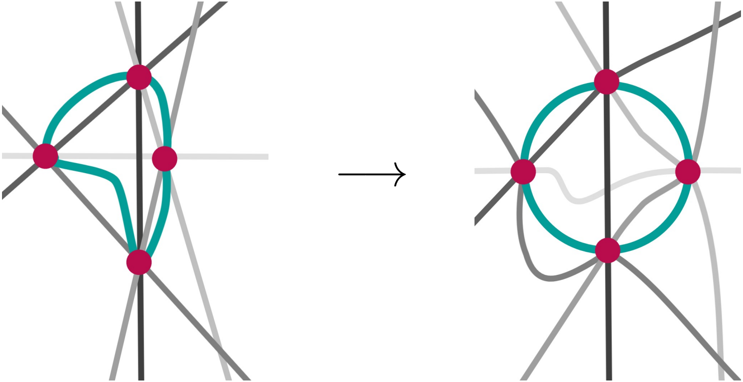


$\longrightarrow$

You can't always go the other way around (Streinu, 05).



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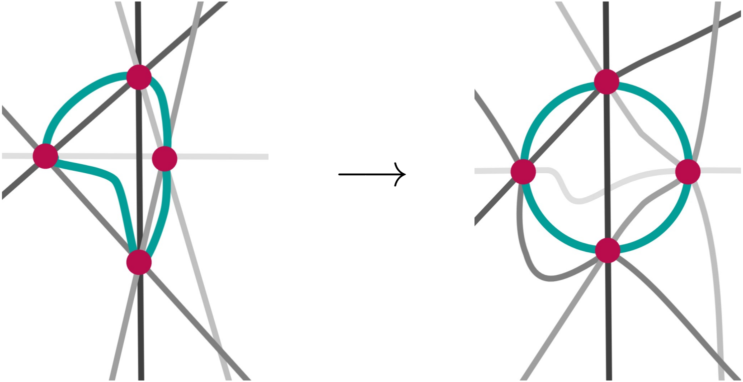

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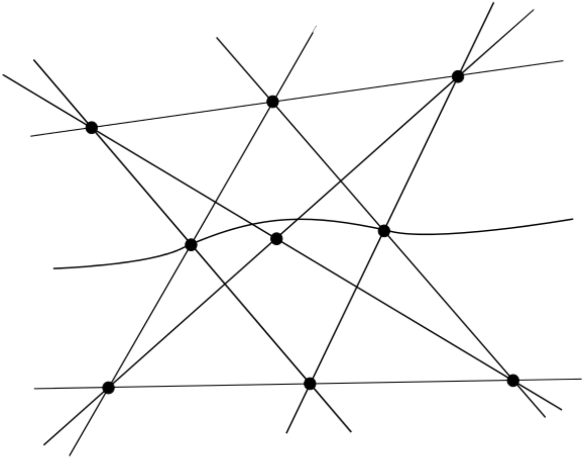

*picture from Wikipedia

Babia Góra

Theorem

We can compute ω and find a $(3 \cdot 4^{\omega-1})$-colouring of an **ordered pseudo-visibility graph** *in polynomial time.*

Proof idea.

1) Find forbidden sub-structures.
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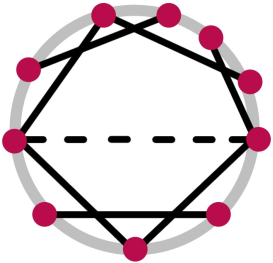

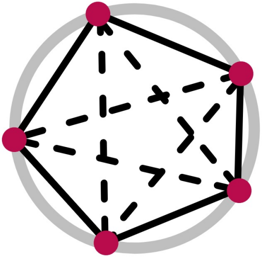

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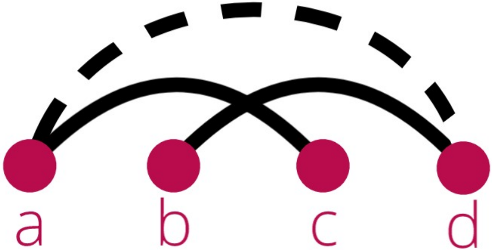

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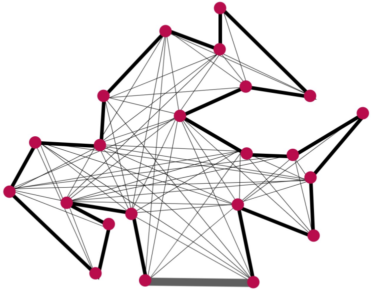

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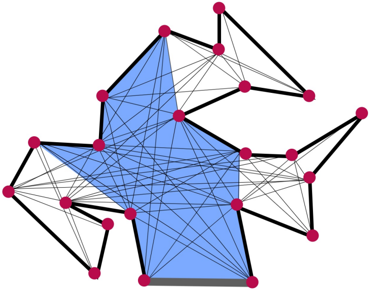

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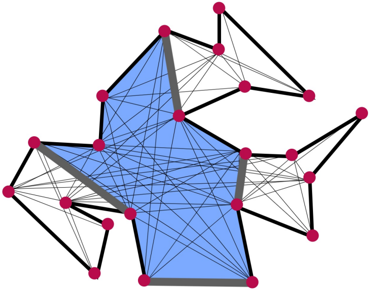

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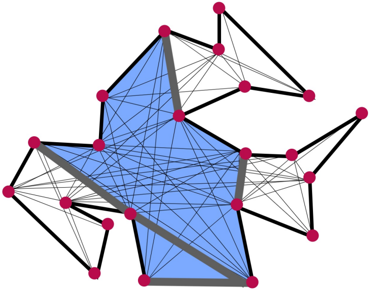

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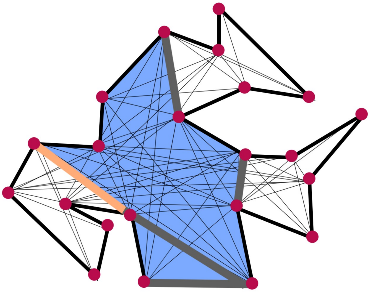

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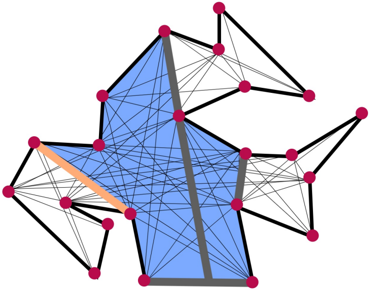

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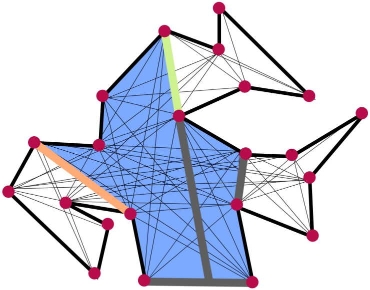

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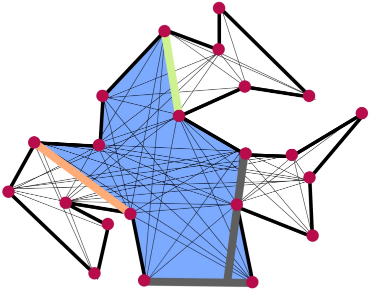

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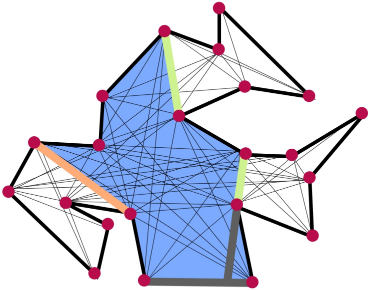

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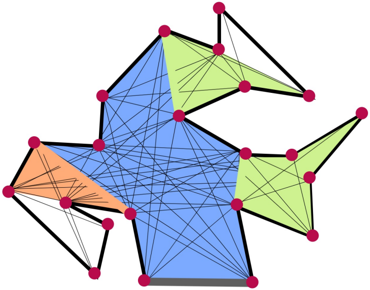

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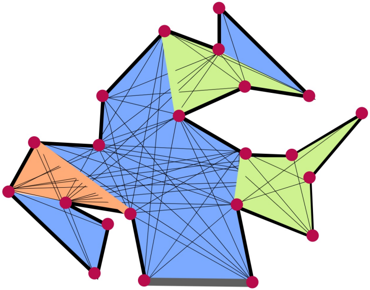

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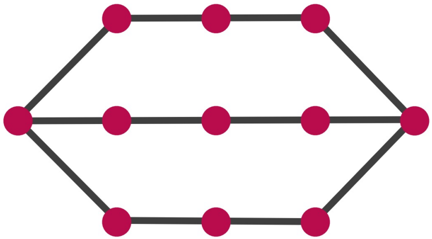

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The recognition problem can be solved in polynomial-time.

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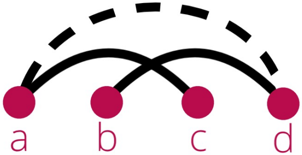

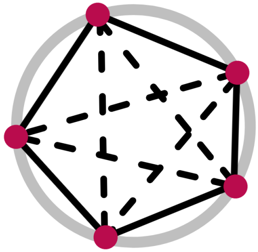

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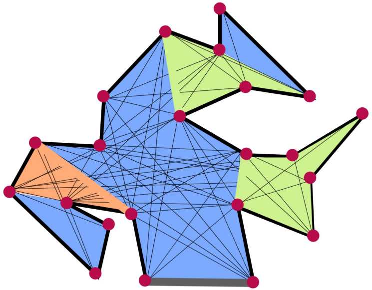

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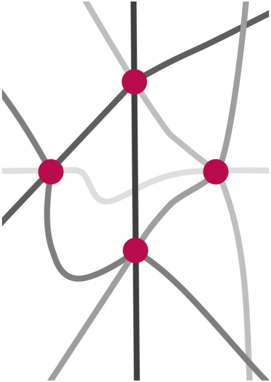

- Hamiltonicity $\longrightarrow$ Jordan curve
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- Needs to satisfy: $\lambda ab \wedge \lambda ac \wedge \lambda ad \wedge \lambda bc \wedge \lambda cd \Longrightarrow \lambda bd$
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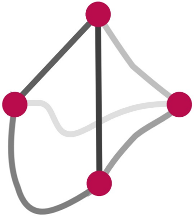

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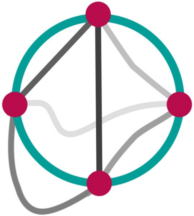

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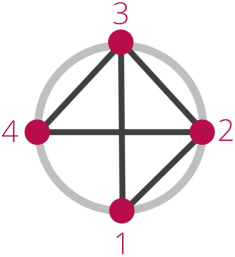

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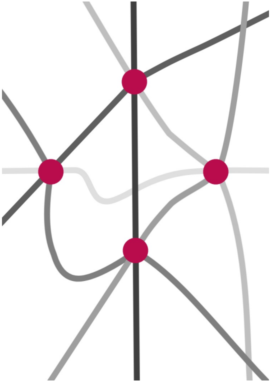

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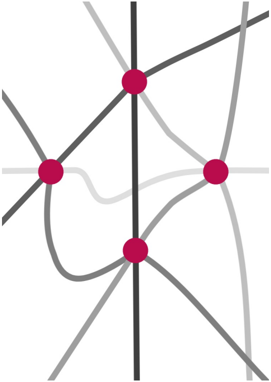

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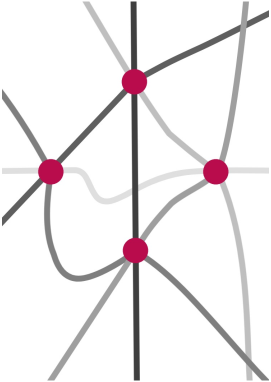

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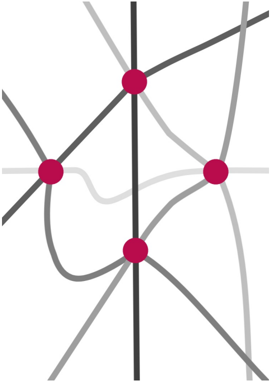

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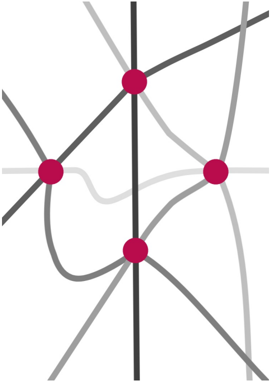

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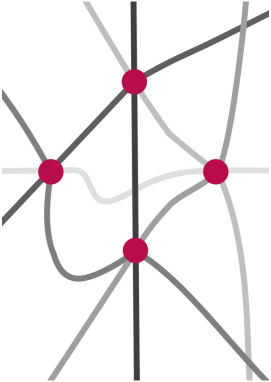

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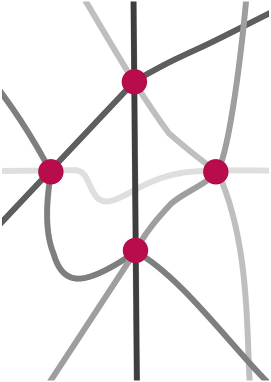

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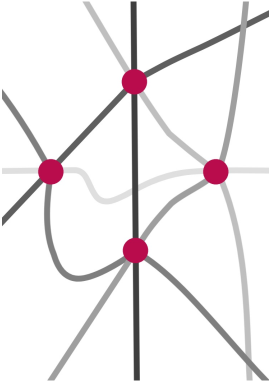

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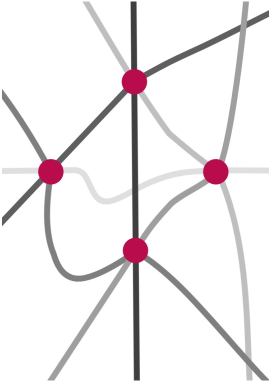

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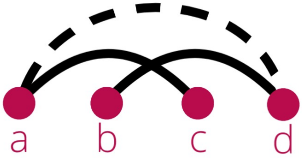

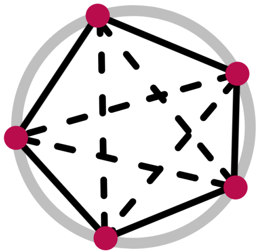

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Recognizing **ordered visibility graphs** *is NP-hard.*

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But recognizing **ordered pseudo-visibility graphs** *is in P.*

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There is a polynomial p so that every capped graph with clique number ω has chromatic number at most $p(\omega)$.

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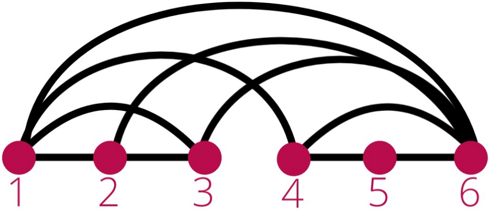

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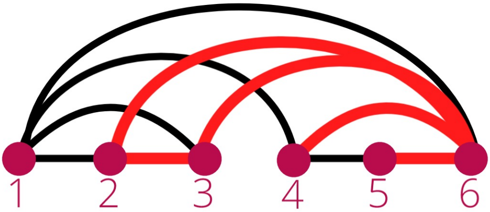

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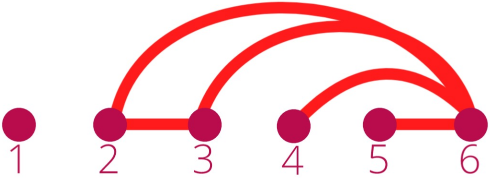

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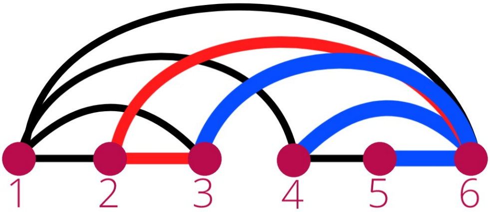

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