Open Problems for the 2026 Barbados Graph Theory Workshop

Collected by Julien Codsi

Problem 1. Coarse Menger for series-parallel graphs (From Paul Seymour)

Let S, T be sets of vertices of a series-parallel graph G. Suppose that there do not exist k S-T paths pairwise at distance at least c. Must there be a set X of at most $k-1$ vertices such that every S-T path in G passes within bounded distance from X? (The bound can depend on k, c but must be independent of G.)

This is not true for graphs in general (see https://www.arxiv.org/abs/2401.06685), not even for graphs of tree-width six when $c = k = 3$. It is true for general graphs if $k = 2$; it might be true for general graphs if $c = 2$, and it might be true for planar graphs with general c, k. Is it true for series-parallel graphs if $k = 3$ and $c = 2$?

Update: This was solved positively for any k, c when $|S \cup T|$ is bounded during the workshop.

Problem 2. A/B **properties** (From Daniel Carter)

Let A and B be graph properties, and say G has the A/B *property* if there exists a partition $P = \{P_1, \dots, P_k\}$ of $V(G)$ so that $G[P_i]$ has property A for all i and G/P (that is, the graph obtained by identifying all the vertices in each part) has property B. For various choices of A and B, what graphs have the A/B property? When is the A/B property testable in polytime?

- If A and B are both the property of being a tree, then testing the A/B property is NP-complete, since a planar triangulation has the tree/tree property iff its dual is Hamiltonian. Same goes for forest/forest, tree/forest, and forest/tree.
- If A is "edgeless" and B is "subgraph of H" then A/B = "H-colorable", which is NP-complete to test unless H is bipartite.

Interesting cases to investigate might be A = "path" or A = "chordal", for various choices of B (bounded size, "path", "forest", ...).

Problem 3. Something about matroids (From Daniel Carter)

Let $S = \{F_1, F_2, \dots\}$ be an infinite collection of finite fields. Is there always a finite subset of S, say S', so that a matroid is realizable over every field in S iff it is realizable over every field in S'? It is easy to prove if you believe that matroids realizable over a given finite field are wqo by minors, but maybe there is another way. It is also true if $\mathbb{F}_2 \in S$ [49] or $\mathbb{F}_3 \in S$ [50]. Recently, Baker–Lorscheid have proved it holds when restricting to matroids with no U_5^2 or U_5^3 minor, generalizing both of the previous results [6]. On the other hand, with a suitable version of Mnëv's universality theorem it is easy to see that it is *not* true if the fields of S are allowed to be infinite; this gives another proof of the fact that matroids representable over any given infinite field are not wqo (even restricted to rank 3 matroids).

Problem 4. Stability version of degeneracy (From Maria Chudnovsky)

Maria: I edited the problem to say what I actually meant

For every positive integer t there is $c(t)$ such that if G is a graph with no induced subgraph isomorphic to $K_{t,t}$ or to a subdivision of a 1-subdivision of K_t, then there is $v \in V(G)$ with $\alpha(N(v)) \leq c(t)$.

If this is too hard, replace the assumption with "no induced minor isomorphic to $K_{t,t}$", or replace $c(t)$ with $\log^{c(t)}(|V(G)|)$.

From Daniel Carter: The original version of this problem said "subdivision of K_t". That version is an easy consequence of the main result of [36], which states that for any H and s, graphs of sufficiently large average degree either (i) contain $K_{s,s}$ as a subgraph or (ii) contain a subdivided H as an induced subgraph. Take $H = K_t$, $s = R(t,t)$, and apply Ramsey's theorem if (i) occurs to find either a large induced sub-K_t or a large induced $K_{t,t}$ in any graph with large average (or minimum) degree.

From Sepehr Hajebi (to Daniel Carter?): [Written before the edition] Right. However, as the title suggests, I suspect the problem really wants to ask whether, for every positive integer t and every graph H, there exists $c = c(t,H)$ such that every graph G with no induced subgraph isomorphic to $K_{t,t}$ or to any subdivision of H has a vertex v with $\alpha(N_G(v)) \leq c$. [This is equivalent to the edited version.]

The idea is to strengthen "polynomial bounds in ω" to "α-boundedness." Here, if true, the above would be a strengthening of the main result of [26]: For every positive integer t and every graph H, there exists $d = d(t,H)$ such that every graph G with no induced subgraph isomorphic to $K_{t,t}$ or to any subdivision of H has a vertex of degree at most $\omega(G)^d$ – which, I guess, is the motivation for the problem.

I think this is open for general H. Even the weakening to "no induced minor isomorphic to $K_{t,t}$," mentioned by Maria, does not seem to follow from [36].

Problem 5. Ideal clutters have bounded chromatic number (From Tony Huynh)

Is there a universal constant c such that every ideal clutter without a hyperedge of size 1 has chromatic number at most c?

Here, a *clutter* is a hypergraph in which no hyperedge is contained in another hyperedge. A clutter is *ideal* if its vertex cover number is equal to its fractional vertex cover number. The above problem was proved for binary ideal clutters with $c = 4$ by Abdi, Cornuéjols, Lee, and myself. It could be tractable in general for some large c. See [1] for more information, where the conjecture was first posed.

Update (Carla, Freddie, Raphael and others): An ideal clutter with at most x sizes of hyperedges has chromatic number at most $x + 1$ (and some more general weighted version holds that I dont want to state). The fractional chromatic number of an ideal clutter equals $s/(s-1)$ where s is the minimum size of a hyperedge. The chromatic number of an n-vertex ideal clutter is at most $O(\log n / \log\log n)$.

Problem 6. χ-boundedness in net-free graphs (From Sepehr Hajebi)

For every positive integer t, the class of all net-free graphs whose triangle-free induced subgraphs have chromatic number at most t is χ-bounded.

There is now a short proof [29] for the same statement with "net-free" replaced by "bull-free" that does not rely on any result about the structure of bull-free graphs. This is what makes me think, although very little seems to be known about the structure of net-free graphs, that the net-free question may also be tractable.

(Recall that the *net* is the graph obtained from a triangle by adding a degree-one neighbor for each vertex of the triangle. The *bull* is the graph obtained from a triangle by adding a degree-one neighbor for two of the vertices of the triangle.)

Problem 7. Triangle-free sun-free graphs (From Sepehr Hajebi)

This is to recall a problem by Nicolas Trotignon from last year (originally posed for χ-boundedness, here restricted to the triangle-free case):

Every triangle-free graph of large enough chromatic number contains an induced sun.

Here, for an integer $t \geq 4$, a *t-sun* is obtained from a t-cycle by adding a degree-one neighbor to each vertex of the cycle. A *sun* is a t-sun for some $t \geq 4$.

This is still open. With Sophie Spirkl, we proved something vexatiously close [32]: every triangle-free graph G with $\chi(G) > 43$ contains either an induced t-sun for some $t \geq 5$, or an induced 4-sun with a single degree-one vertex deleted. The proof avoids most standard χ-boundedness techniques, and I'm not sure how to extend it.

Using more conventional tools, I can prove the 2-controlled case. I'd be very happy to exchange ideas on the non-2-controlled case (i.e., where G has huge chromatic number but all radius-2 balls have small chromatic number).

Problem 8. On (tw, ω)-boundedness and (pw, ω)-boundedness (From Sepehr Hajebi)

A graph class $\mathcal{G}$ is (*polynomially*/*linearly*) (tw, ω)-*bounded* if there is a (polynomial/linear) function $f : \mathbb{N} \to \mathbb{N}$ such that $\mathrm{tw}(G) \leq f(\omega(G))$ for all $G \in \mathcal{G}$. Define (polynomial/linear) (pw, ω)-boundedness analogously. I'd like to propose a few problems on these notions:

(1) *Is it true that for every hereditary linearly (tw, ω)-bounded class $\mathcal{G}$, there exists $c \in \mathbb{N}$ such that every graph in $\mathcal{G}$ is the intersection graph of a collection of connected subgraphs of a graph with treewidth at most c? What if $\mathrm{tw}(G) \leq \omega(G) + d$ for some $d \in \mathbb{N}$ and all $G \in \mathcal{G}$?*

It is easy to see that if G is the intersection graph of a collection of connected subgraphs of a graph with treewidth at most c, then $\mathrm{tw}(G) < (c+1)\,\omega(G)$. So, (1) is asking whether the converse holds (and I hope the answer is "no!"). Even the weaker conclusion that "graphs in $\mathcal{G}$ have bounded tree independence number" is open. Somewhat perversely, the analogous statement for pathwidth is false (usually it goes the other way): There is a neat construction [5] of a hereditary class $\mathcal{G}$ with unbounded "path independence number" where $\mathrm{pw}(G) \leq 2\,\omega(G)$ for every $G \in \mathcal{G}$.

Update. Rose McCarty found a proof that the answer is indeed "no." More strongly, Rose shows that for some $c > 0$, there is a hereditary class $\mathcal{G}$ such that every graph $G \in \mathcal{G}$ satisfies $|V(G)| \leq c\,\omega(G)$, and yet for every $d > 0$, there is a graph in $\mathcal{G}$ that is not the intersection graph of a collection of connected subgraphs of a graph of treewidth at most d.

(2) *Is it true that for every chordal graph H and every hereditary class $\mathcal{G}$ of H-free graphs, if $\mathcal{G}$ is (tw, ω)-bounded, then $\mathcal{G}$ is polynomially (tw, ω)-bounded?*

The choice of H would be best possible: It is proved in [15] that if H is not chordal, there is a hereditary class of H-free graphs that is (tw, ω)-bounded but not polynomially so. Some known things (the first two are easy though not written down): (a) True for H if and only true for every component of H; (b) True when H is a subdivided star; (c) True when H is a tree, but excluded as an induced minor [30] (in this case, pathwidth is polynomially bounded by ω); (d) True when H is a tree, and graphs in $\mathcal{G}$ are theta-free [13]. What about the natural common strengthening of (b) and (c): "True when H is a tree and all subdivisions of H are excluded as induced subgraphs"? What if H is the diamond? What if H is a general chordal graph, but excluded as an induced minor?

(3) *Is every hereditary* (pw, ω)*-bounded class linearly* (pw, ω)*-bounded?*

It was proved recently that every hereditary (pw, ω)-bounded class is *polynomially* (pw, ω)-bounded [30]. I would very much like to believe that the answer to (3) is "yes" (pathwidth has always been kind to us!). That said, at least to me and for now, extending the ideas from [30] to obtain linear bounds seems completely hopeless. Here is a very modest step in this direction that remains open: For every $r \in \mathbb{N}$, there exists $c \in \mathbb{N}$ such that if G has maximum degree $\Delta \in \mathbb{N}$ and no induced minor isomorphic to the radius-r binary tree, then $\mathrm{pw}(G) \leq c\,\Delta$ (or at least $\mathrm{tw}(G) \leq c\,\Delta$).

Problem 9. Induced saturation for even cycles (From Sepehr Hajebi)

For every integer $t \geq 2$, there is a graph G that is C_{2t}-free, but such that adding or deleting any edge in G creates an induced copy of C_{2t}.

This is known for only finitely many values of t (I know constructions for $t \in \{2, 3, 4, 5\}$). Such a graph G is also known to exist if we require only that adding any edge creates an induced C_{2t} [47] (with G not complete, to avoid the vacuous case), and also if we require only that deleting any edge creates an induced C_{2t} [25] (with G not edgeless). The former construction is quite neat; the latter decidedly less so. It is somewhat strange that the analogous statement for odd cycles is easy [7]: For every $t \geq 2$, the line graph of $K_{t+1,t+1}$ is C_{2t+1}-free, yet adding or deleting any edge creates an induced C_{2t+1}. (See also Problem 15 by Carla.)

Problem 10. k-tuples in matroid circuits (From Jim Geelen)

In his paper *triples in matroid circuits*, Seymour proves that, if X is a set of at least 3 elements in a vertically 4-connected binary matroid and there is no circuit that contains at least 3 elements of X, then M is the cycle-matroid of a graph G in which all edges in X are incident with a common vertex.

Conjecture: *For an integer $k \geq 3$, if X is a sufficiently high-rank set of elements in a binary matroid M with sufficiently large vertical-connectivity and no k elements of X are contained in a circuit, then there is a rank-f_k projection M' of M such that M' is the cycle-matroid of a graph G in which all edges in X are incident with a common vertex.*

Here a *rank-f_k projection* is obtained by adding f_k new elements to the matroid and then contracting them.

There is a natural extension of this conjecture to all matroids, which involves "frame matroids" instead of graphic matroids. There is also a natural extension of the conjecture in which we drop the connectivity requirement and instead ask for the structure relative to a tangle.

From Paul Seymour Do you mean if no circuit contains k elements of X? (yes.)

Problem 11. Partitioning with respect to treewidth (From David Wood and Kevin Hendrey)

We asked the following question in [34]. For integers $p, q \geqslant 1$, let $f(p, q)$ be the minimum integer such that every graph with treewidth at least $f(p, q)$ has pairwise disjoint connected subgraphs $G_1, G_2, \ldots, G_n$ such that each G_i has treewidth at least p, and contracting each G_i to a vertex gives a graph with treewidth at least q. It follows from the best known bounds in the Grid-Minor-Theorem [16] that $f(p,q) \in O^*((pq)^9)$. Are there better upper bounds on $f(p,q)$? Perhaps $f(p,q) \in O(pq)$? Perhaps random graphs provide lower bounds? Chekuri and Chuzhoy [10] have results about partitioning a graph with large treewidth into subgraphs with large treewidth, but without the contraction property.

Problem 12. Vertex-minor Ramsey (From Caleb McFarland)

Define the *vertex-minor Ramsey number* $R_{vm}(k)$ to be the smallest n such that any graph on n vertices contains an independent set of size k as a vertex-minor. Because a clique of size $k+1$ contains an independent set of size k as a vertex-minor, clearly $R_{vm}(k) \leq R(k, k+1)$ where $R(s,t)$ is the usual Ramsey number. In a current project [4], we conjecture that $R_{vm}(k)$ is polynomial in k. We prove that with high probability, $\mathbb{G}(n, 1/2)$ contains every graph on $\Omega(\sqrt{n})$ vertices as a vertex-minor. In particular, $\mathbb{G}(n, 1/2)$ contains an independent set of size $\Omega(\sqrt{n})$ as a vertex-minor, and so is not a counterexample to $R_{vm}(k) \in \text{poly}(k)$. We are able to get bounds of $\Omega(k^2) \leq R_{vm}(k) \leq 2^k - 1$ which improves the trivial bound of $R(k, k+1) \leq 3.8^{k+o(k)}$ [28]. Even showing $R_{vm}(k) = 2^{o(k)}$ would be interesting.

Note that if you make the problem easier by restricting to any proper vertex-minor closed class, then you now have the Erdős-Hajnal property [14], and so every graph in the class contains an independent set of size $\Omega(n^{1/c})$ as a vertex-minor for some constant c possibly depending on the class.

Problem 13. η-boundedness for graphs with bounded rank-width (From Xiying Du and Rose McCarty)

For a graph G, $\eta(G)$ denotes the minimum size of a vertex set in G that intersects every maximum independent set. A hereditary class $\mathcal{C}$ is η-bounded if there exists a function f such that $\eta(G) \leq f(\omega(G))$ for all $G \in \mathcal{C}$ [31].

This parameter relates to a packing/covering phenomenon. In graphs with independence number linear in the number of vertices, one cannot pack many disjoint maximum independent sets. It is then natural to ask when it is possible to *cover* all maximum independent sets with a bounded number of vertices.

We ask the following:

Are classes of bounded rank-width η-bounded?

We believe we can prove that η-boundedness is preserved under substitution and 1-join operations.

Problem 14. Reconfiguring nowhere-zero flows (From Louis Esperet)

Let us say that that two nowhere-zero k-flows (or A-flows, for some abelian group A) of some graph G are *adjacent* if the support of their difference is a cycle (i.e. a 2-regular connected graph). In other words, one flow is obtained from the other by adding some flow value along a cycle.

Two nowhere-zero k-flows (or A-flows) f_1 and f_k of G are *connected* if there is a sequence $f_2, \ldots, f_{k-1}$ of nowhere-zero k-flows (or A-flows) of G such that any two consecutive flows in the sequence $f_1, \ldots, f_k$ are adjacent.

With Aurélie Lagoutte and Margaux Marseloo [23], we asked the following: *Is it true that there is an integer k, such that for every 2-edge-connected graph G, all nowhere-zero k-flows of G are connected?* (we conjecture that this holds for $k = 5$). Similarly, *is there an abelian group A such that for every 2-edge-connected graph G, all nowhere-zero A-flows of G are connected?* (we conjecture that this holds for $A = \mathbb{Z}_5$).

Update 1: I (Louis) think we might have a counterexample (found with Clément Legrand) for the cases $k = 5$ and $A = \mathbb{Z}_5$ of the problems above. The ideas do not work at all for $k \geq 6$ and $|A| \geq 6$.

Update 2: Raphael found a (much simpler) conterexample for $k = 5$ and $A = \mathbb{Z}_5$ with his colleagues Patryk Morawski and Yuval Wigderson.

Update 3: the group question was solved positively for $A = \mathbb{Z}_2^k$ (for some large but constant k) during the workshop.

The integer question remain open for $k \geq 6$.

Problem 15. More induced saturation (From Carla Groenland)

It has turned out to be very difficult to classify for which graphs H there exists an H-induced-saturated graph. For example, the existence of such a graph is not known for most even cycles (see Sepehr's Problem 9), and we also do not know the answer to the following problem.

Are there infinitely many graphs H with an edge and a non-edge, for which there does not exist an H-induced saturated graph?

As far as I am aware, only one such example is known so far, namely P_4.

A graph G is called H-induced-saturated if G does not contain H as an induced subgraph, but adding or removing any edge to G creates an induced copy of H. Let $\mathcal{G}_{H,n}$ be the graph with the n-vertex H-free graphs are vertices and an edge between G and G' whenever they have edit-distance 1, that is, $|E(G) \triangle E(G')| = 1$. An H-induced-saturated graph then corresponds to an isolated vertex. Can we obtain a characterisation for a weaker requirement?

For which graphs H is $\mathcal{G}_{H,n}$ disconnected for all sufficiently large n?

The problems above are posed in joint work with Marthe Bonamy, Tom Johnston, Natasha Morrison and Alex Scott (https://arxiv.org/pdf/2506.08810) and the second is based on discussions with Torsten Ueckerdt.

Problem 16. χ-boundedness and polynomial χ^*-boundedness (From Tung Nguyen)

> *Let $\mathcal{G}$ be a χ-bounded hereditary class of graphs such that there exists $d \geq 1$ for which every graph $G \in \mathcal{G}$ has fractional chromatic number at most $\omega(G)^d$. Is $\mathcal{G}$ polynomially χ-bounded?*

This question likely was asked somewhere (as a weakening of Louis's false conjecture) but I can't find a reference. If the answer is affirmative then the general polynomial χ-boundedness problem would be reduced to its "pure" χ-boundedness version and its fractional relaxation. In particular this would imply that the family of forests satisfying polynomial Gyárfás–Sumner is closed under disjoint union, since the same is known for the fractional version of polynomial GS [40, Section 5].

A closely related problem which also relates to a question of Sang-il last year (here $\chi^\star$ denotes fractional chromatic number):

> *Let $\mathcal{G}$ be a (polynomially) $\chi^\star$-bounded hereditary class of graphs such that all triangle-free graphs in $\mathcal{G}$ have bounded chromatic number. Is $\mathcal{G}$ (polynomially) χ-bounded?*

Problem 17. What is the truth for Hajós? (From Raphael Steiner)

Hajós' (famously false) strengthening of Hadwiger's conjecture stated that every graph G which does not contain a subdivision of K_t satisfies $\chi(G) < t$. Catlin disproved this, and later

Erdős and Fajtlowicz, by considering the random graph $G(n, 1/2)$, showed a lower bound of $\Omega(t^2/\log t)$ on the chromatic number of graphs with no K_t-subdivision.

A well-known result, independently due to Bollobás-Thomason and Komlós-Szemerédi, states that the average degree of a graph with no K_t-subdivision is $O(t^2)$, and this is tight.

Hence, we obtain an upper bound $O(t^2)$ and a lower bound $\Omega(t^2/\log t)$ for the maximum chromatic number of a graph with no K_t-subdivision. Fox, Lee and Sudakov conjectured in 2012 that the truth lies with the lower bound, and proved this when the graph G under consideration has $O(t^2)$ vertices.

Can we at least show a bound of $o(t^2)$ for the chromatic number? Intuitively one may wish to take inspiration from the recent advances on Linear Hadwiger's conjecture that show that the core of the problem there lies with small graphs. A similar result for subdivisions seems plausible.

Problem 18. A problem inspired by Vu's conjecture (From Linda Cook)

These questions are joint with with Ross Kang, Eileen Robinson and Gabriëlle Zwaneveld and appear in [17]

The codegree of two distinct vertices x, y is there number of common neighbors and and the maximum codegree between any two distinct vertices of a graph G is denoted by $\Delta_2(G)$. Vu (2002) proposed that, provided $\Delta_2(G)$ is not too small as a proportion of the maximum degree $\Delta(G)$ of G, the chromatic number of G should never be too much larger than $\Delta_2(G)$. We proved that, in the case where G is claw-free, the chromatic number of G is indeed at most $\Delta_2(G) + 3$ (which is tight). Inspired by this we ask that what other graph classes yield such a bound.

Question 1. *Within which graph hereditary classes $\mathcal{G}$ is there a constant C so that $\chi(G) \leq \Delta_2(G) + C$. for every graph in $\mathcal{G}$?*

Note that since $\omega - 2 \leq \Delta_2$ if $\mathcal{G}$ is χ-bounded by a function of the form ω plus a constant then it trivially has this property.

For the second question I asked, I just give you my handwritten notes sorry. (Figure 18).

Problem 19. Graphs with linear domination width ✓ (From Julien Codsi)

The *tree domination width* of a graph G is the minimum integer k such that G admits a tree decomposition $(T, \mathcal{B})$ in which every bag $B \in \mathcal{B}$ satisfies $\gamma(G[B]) \leq k$, where γ denotes the domination number. Equivalently,

$$\mathrm{tdw}(G) = \min_{(T,\mathcal{B})} \max_{B \in \mathcal{B}} \gamma(G[B]).$$

For which function f are there graph families where $\mathrm{tdw} = \Omega(f(n))$?

Obviously $\Omega(\sqrt{n})$ is achievable by a grid. More interestingly, $G(n, p)$ has linear treewidth almost surely when $p = c/n$ for some fixed $c > 1$ (see [37]). In this regime, the maximum degree is with high probability less than $\log(n)$ (can be improved with some log logs). Therefore, to dominate a linear-size bag, one needs at least $\Omega(n/\Delta) = \Omega(\frac{n}{\log(n)})$. Is it possible to get rid of this log with a more clever construction?

Raphael: I believe a random regular graph G(n,d) with d a constant at least 3 will work. They do not have sublinear balanced separators (as should follow from some citable literature results on their expansion properties), so have linear treewidth. For example this should do.

Julien: Very nice! I wasn't aware of that family. Solved!

Problem 20. Intersection spectrum of hypergraphs with large c-strong chromatic number (From Freddie Illingworth)

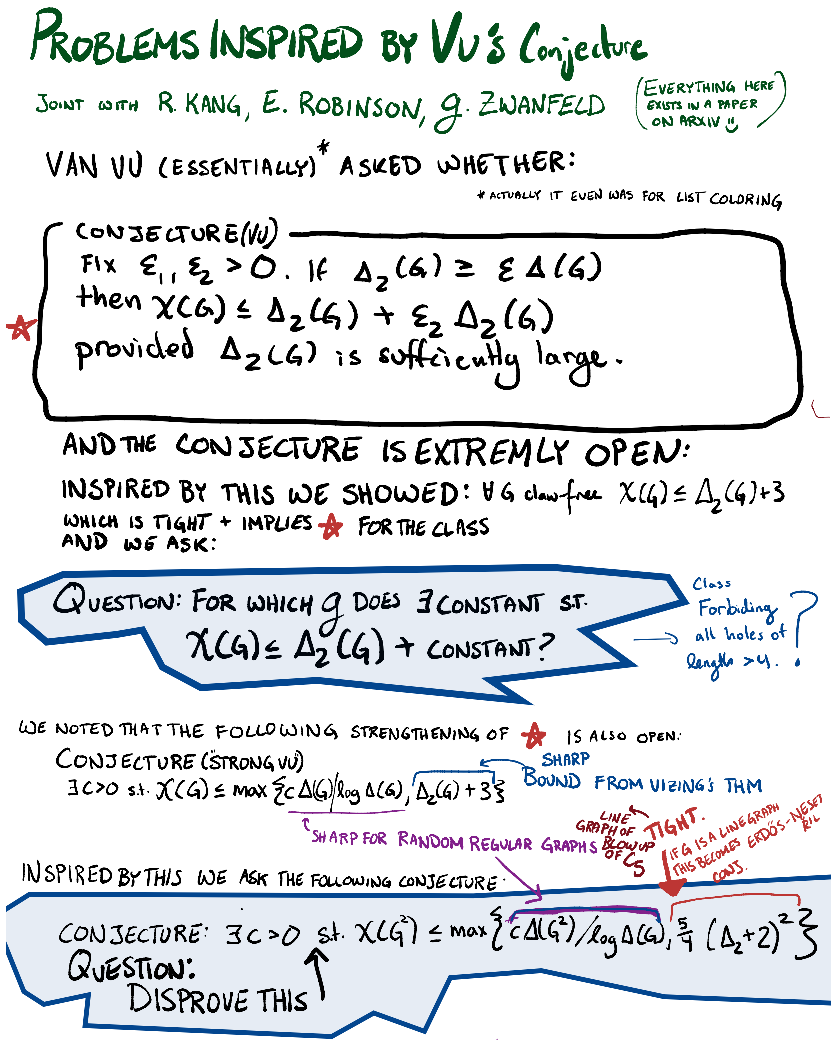

Figure 1: Question 2 for 18

A c-strong colouring of a hypergraph $\mathcal{H}$ is a colouring of the vertices of $\mathcal{H}$ such that every edge is either (i) rainbow or (ii) receives at least c different colours. When $c = 2$ this is the usual definition of colouring and when $c = \infty$ this is the usual definition of strong colouring. $\chi(\mathcal{H}, c)$ is the c-strong chromatic number of $\mathcal{H}$: the smallest number of colours needed in a c-strong colouring of $\mathcal{H}$.

The intersection spectrum of a hypergraph $\mathcal{H}$ is $I(\mathcal{H}) := \{|e \cap f| : e, f \in E(\mathcal{H}), e \neq f\}$. Classical work of Erdős and Lovász [21] shows that (a) if $\chi(\mathcal{H}, 2) \geqslant 3$, then $0 \in I(\mathcal{H})$ and (b) if $\chi(\mathcal{H}, 2) \geqslant 4$, then $1 \in I(\mathcal{H})$.

In [33], Kevin Hendrey, Nina Kamčev, Jane Tan, and I showed that there is a function f such that if $\chi(\mathcal{H}, c) \geqslant f(c)$, then the smallest element of $I(\mathcal{H})$ is at most $c - 2$. This corresponds to (a) when $c = 2$. Does a similar result hold for (b)? That is,

> *Is there a function g such that if $\chi(\mathcal{H}, c) \geqslant g(c)$, then the smallest non-zero element of $I(\mathcal{H})$ is at most $c - 1$?*

$c - 1$ would be best possible (take any (hyper)graph with large chromatic number and replace each vertex by a set of $c - 1$ vertices). In fact, I do not know if there are functions g and h such that if $\chi(\mathcal{H}, c) \geqslant g(c)$, then the smallest non-zero element of $I(\mathcal{H})$ is at most $h(c)$.

For some papers on these problems that may have useful ideas, see [9, 3].

Problem 21. Deciding branch-width at most k for small input instances (From Sang-il Oum)

A connectivity function on a finite set V is a symmetric submodular function $f : 2^V \to \mathbb{Z}$ with $f(\emptyset) = 0$. Suppose that $|V| = n$ and $n \leq 3^{4k}$. Is it possible to decide whether the branch-width of f, given by an oracle, is at most k in time $2^{O(k)}$?

If true, then as a corollary of my recent theorem with Tuukka Korhonen [35], we will have a fixed-parameter tractable algorithm to find a branch-decomposition of f of width at most k, if it exists, in time $2^{O(k)} n^6 \log^2 n$, when f is given by an oracle taking the unit time for each query.

Currently, we use the algorithm of Oum and Seymour [43] for such small cases $n \leq 3^{4k}$. This gives the running time $2^{O(k^2)}$ when $n \leq 3^{4k}$. If we use the exact exponential-time algorithm [42], we would get $O^*(2^{3^{4k}})$.

Problem 22. A directed relative of Thomassen's conjecture (From Raphael Steiner)

It is a well-known fact that every graph G with minimum degree $\delta(G) \geq 2k - 1$ has a bipartite subgraph G' with $\delta(G') \geq k$.

The same is false for digraphs, even qualitatively. Indeed, as we observe in [12], an old construction of Thomassen implies that there are digraphs of arbitrarily large minimum out-degree where every bipartite subgraph has minimum out-degree 0.

But we conjecture in the same paper that for every $k, g \in \mathbb{N}$ there exists some $d = d(k, g)$ such that every digraph D with minimum out-degree $\delta^+(D) \geq d$ has a subdigraph D' with $\delta^+(D') \geq k$ which is "locally bipartite" in the sense that the odd-girth of the underlying graph of D' is at least g. This is the open problem I propose. In [12] we prove this for $g = 7$.

Problem 23. Ramsey perfect graphs (From Jane Tan)

Is it true that for every perfect graph G, there exists a perfect graph H such that for every 2-edge-colouring of H there is an induced subgraph isomorphic to G that is monochromatic?

This is a question of Nešetřil from at least as early as 1991, who remarks that a negative answer is expected. I don't know of much literature directly on it, but keywords for surrounding work include the Galvin Ramsey property, and induced Ramsey numbers.

Here is a slightly weaker version: Is it true that for every perfect graph G, there exists a perfect graph H such that for every 2-edge-colouring of H there is an induced copy of G in either the red subgraph or blue subgraph.

Daniel Carter: I believe I can find a perfect host graph such that all 2 edge colorings contain a monochromatic $K_{\underbrace{1,1,\ldots,1}_{t},t}$. Raphael said that bipartite graphs are known (using a bipartite host). I can also do the bull graph.

A nice lemma (maybe interesting in its own right) is the following: Let $\mathcal{C}$ be a class (not necessarily hereditary) that is closed under substitution. Then for every $H \in \mathcal{C}$ and $\varepsilon > 0$, there exists a $G \in \mathcal{C}$ such that for every subset $A \subseteq V(G)$ with $|A| \geq \varepsilon|V(G)|$, $G[A]$ contains an induced H. (How to do it: take $G = H^d$ for $d > \frac{\log(1/\varepsilon)}{\log(\frac{n}{n-1})}$ where $n = |V(H)|$, i.e. start with H, substitute a copy of H at every vertex, and repeat many times.)

Using this, I can at least prove that in any hereditary class $\mathcal{C}$ closed under substitution, for every *forest* $F \in \mathcal{C}$ and integer $k \geq 1$, there exists $G \in \mathcal{C}$ such that in every k-edge-coloring, one of the colors contains an induced F (this is the weaker version of the problem). This doesn't help for this problem since bipartite graphs were already known.

Problem 24. Are induced-minor-closed families small? (From Maria Chudnovsky)

It was shown in [41] that for every proper minor-closed class I of graphs there exists a constant c such that for every integer n the class I includes at most $n!cn$ graphs with vertex-set $\{1, 2, \ldots, n\}$. Is the same true for induced-minor-closed classes?

From Tung: I think the answer is no. The class of string graphs is closed under taking induced minors and it was shown by Pach and Tóth [44, Theorem 1] that the number of n-vertex labelled string graphs is at least $2^{\frac{3}{4}\binom{n}{2}}$.

From Raphael: I think another example is the class of chordal graphs. They are also closed under induced minors. Every split graph is chordal, and it is easy to see that the number of split graphs is at least the number of bipartite graphs with both sides of size $n/2$, which is $2^{(1-o(1))n^2/4}$.

Problem 25. Vertex-disjoint cycles with distinct lengths in digraphs (From Tony Huynh)

Conjecture 1. Is there a function $f(k)$ such that every digraph with minimium out-degree at least $f(k)$ contains k vertex-disjoint directed cycles with distinct lengths?

This is a conjecture of Lichiadopol from 2014. Bensmail, Harutyunyan, Le, Li, and Lichiardopol [8] proved the corresponding result for graphs with minimum degree at least $\frac{5k^2+5k-2}{2}$.

Rather than a sufficient condition, one can instead ask if the following Erdős–Pósa result holds.

Conjecture 2. Are there functions $f_1(k)$ and $f_2(k)$ such that every digraph D either contains k vertex-disjoint directed cycles with distinct lengths or a set of at most $f_1(k)$ vertices X such that $D - X$ contains at most $f_2(k)$ directed cycle lengths?

This is a conjecture of Gollin, Gorsky, McFarland, Muzi, Wiederrecht, and myself, where we prove the analogous result for graphs with $f_1(k) = O(k^9)$ and $f_2(k) = k - 1$. The fact that we can take $f_2(k) = k - 1$ was done here at Barbados with Kevin Hendrey.

Update. Kevin Hendrey and Meike Hatzel showed that Conjecture 2 is false, but the half-integral version might still hold. For graphs, Paul Wollan found a cute proof (cuter than what I presented) that we can take $f_1(k) = O(k^7)$ and $f_2(k) = k - 1$.

Problem 26. Sparse Szemerédi's theorem (From James Davies)

For a set of integers A and positive integer k, we let $F_k(A)$ be the collection of k-term arithmetic progressions in A. Note that Van der Waerden's theorem is equivalent to saying that the hypergraph $G = (\mathbb{Z}, F_k(\mathbb{Z}))$ has infinite chromatic number. The following two ("induced" and "non-induced") sparse versions of Van der Waerden's theorem and Szemerédi's theorem were proved around the same time.

Theorem 1 ([45]). *For every $c, k, g \geq 3$ there exists some $A \subseteq \mathbb{Z}$ such that the hypergraph $G = (A, F_k(A))$ has chromatic number at least c and girth at least g.*

Theorem 2 ([46]). *For every $k, g \geq 3$ and $\epsilon > 0$, there exists some $A \subseteq \mathbb{Z}$ and $F \subseteq F_k(A)$ such that the hypergraph $G = (A, F)$ has and girth at least g and $\alpha(G) \leq \epsilon|A|$.*

Clearly these two nice theorems should be one nice theorem.

Conjecture 1. *For every $k, g \geq 3$ and $\epsilon > 0$, there exists some $A \subseteq \mathbb{Z}$ such that the hypergraph $G = (A, F_k(A))$ has and girth at least g and $\alpha(G) \leq \epsilon|A|$.*

Problem 27. Rooted Minor Density Versus Minor Density (From The Old Fart)

We let $mad(H)$ to be the infinimum of the c such that every graph G with $c|V(G)|$ edges contains G as a minor. We let $Rmad(H)$ (the R is for rooted) be the infinum of the c such that for every vertex v of H, for every vertex w of a connected graph G with $Rmad(H)|V(G)|$ edges, there is a minor of H in G so that w is in the image of v.

When is $RMad(H) = Mad(H)$ (true for all vertex transitive graphs but not for any 2-connected graph to which we add a leaf)? For what f is $Rmad(H) < f(mad(H))$. In particular is this true for $f(x) = x + c$ for some contstant c?

Update. Sergey and Jane have a proof that $f(x) = x + O(\sqrt{x})$ suffices.

Problem 28. Weak diameter colorings of bounded treewidth graphs (From Michał Pilipczuk)

There is a nice lemma due to Dvořák and Norin [20, Lemma 10] (see also https://www.mimuw.edu.pl/~mp248287/tw-diam-col.pdf) saying the following: There is a function $f : \mathbb{N} \to \mathbb{N}$ such that for every graph G of treewidth at most k, the vertices of G can be colored with 2 colors so that for any two vertices u, v that can be connected by a monochromatic path (in particular, u and v must have the same color), we have $\text{dist}_G(u, v) \leq f(k)$. (In other words, every monochromatic connected subgraph of G has weak diameter at most $f(k)$, where the weak diameter is the largest distance measured in G between two vertices of the subgraph.) The proof is an induction on the treewidth and gives an upper bound of $f(k) \leq k^{\mathcal{O}(k)}$. Can we prove a better upper bound, ideally polynomial in k?

Problem 29. Finite existentially complete triangle-free graphs (From Daniel Carter)

Let Φ be a first-order graph property. It is well-known that the random graph $G(n, 1/2)$ satisfies Φ with high probability if and only if the Rado graph satisfies Φ. There is a "triangle-free universal graph" called the Henson graph. Does there exist a random triangle-free graph construction such that the set of first-order properties satisfied by the construction with high probability are the same as the properties satisfied by the Henson graph? Several "obvious" random triangle-free graphs don't work:

- A uniformly random triangle-free graph on n vertices is bipartite with high probability, so doesn't contain C_5, unlike the Henson graph.
- Graphs constructed by the triangle-free process don't contain any dense subgraphs with high probability, for instance they don't contain $K_{t,t}$ for sufficiently large t, unlike the Henson graph.

More specifically, I want a triangle-free graph construction (random or not) so that for every pair of disjoint vertex sets A, B where $|A| + |B| \leq k$ and A is a stable set, there is a vertex that is complete to A and anticomplete to B (if considering a random construction, this should hold with high probability). The question of whether there is such a graph was originally raised by Cherlin [11]; currently no examples are known for $k \geq 4$, and it is expected that the answer is that there is no such graph for k sufficiently large [24].

Problem 30. Clustered 3-coloring of (path+$2K_1$)-free apex minor free graphs (From Vida Dujmović)

Esperet and Joret [22] showed that planar graphs with bounded maximum degree are 3-colorable with bounded clustering. Recently, Dvořák [19] greatly generalized it as follows. Let $P_t^{''}$ be a graph obtained by $K_{2,t}$ by adding a path through the t vertices in one bipartition. Dvořák showed that graphs of bounded genus that are $P_t^{''}$-free are 3-choosable with bounded clustering.

Excluding $P_t^{''}$ as a subgraph is necessary as there are planar graphs with such subgraphs that have a clustered chromatic number equal to 4. The result cannot be generalized to all proper minor closed families since a triangular grid plus an apex vertex has no K_6 minor and does not have $P_t^{''}$ for $t \geq 9$, and yet it requires 4 colours thanks to the hex-lemma.

What about the clustered 3-choosability of $P_t^{''}$-free apex minor free graphs? What about clustered 3-colouring? Not even known for clustered 3-coloring of $K_{2,t}$-free apex minor free graphs. 4-colouring is known as proved by Liu, Wood [39] via a very long proof of a much stronger result, and a direct, simpler proof by Dujmović, Esperet, Morin, Wood [18].

In addition to the example above (hex-grid+apex vertex), another indication that this may be true is that bounded treewidth $K_{2,t}$-free graphs are clustered 3-choosable [38].

Problem 31. Geodesic separator for planar graphs (From Julien Codsi)

Is it true that every planar graph has a balanced separator (where no remaining component has more than half the number of vertices) that is the union of two shortest paths? It is known that 3 are enough [48]. Actually, the proof in [48] gives something stronger. There exists a tree decomposition where every bag is the union of 3 geodesics and where the intersection of every bag is the union of 2 geodesics. Therefore, if we relax the definition of balanced to "no component has more then 2/3 of the vertices" then the answer is yes. More generally, for any forbidden minor H, it is known that there exists a balanced separator with c_H shortest paths [2]. Unfortunately, in [2], c_H is quite large as it depends on the "almost embeddability into surfaces" of graphs with forbidden minors (from graph minor XVI). Could it be that $|E(H)|$ shortest paths are enough? Can we get a good lower bound on the number of paths required? All I know is that, in general, $|V(H)|/4$ is not enough.

Problem 32. Polygon visibility graphs (From James Davies)

Are polygon visibility graphs polynomially χ-bounded?

Problem 33. Restricted induced Ramsey for the triangle (From Daniel Carter)

Is there a $\{K_6, \text{odd hole}\}$-free graph G such that all 2-edge-colorings of G contain a monochromatic triangle? (If so, what about $\{K_4, \text{odd hole}\}$-free?) Note that there are K_6-free graphs, such as the complement of the $(7, 2)$ generalized Petersen graph and the complement of $C_5 + 3K_1$ (and there are larger examples that are K_4-free). See [27].

Problem 34. Spread/width trade-off for tree decompositions of the balanced grid (From Carla Groenland)

David Wood introduced the notion of the spread of a vertex $v \in V(G)$ in a tree decomposition $(T, (B_t)_{t\in V(T)})$ in https://arxiv.org/pdf/2509.01140 as the number of bags that contain v.

Wood in particular made the conjecture that in every tree-decomposition of the $n \times n$ grid with width n, some vertex has spread $\Omega(n)$. With Hans Bodlaender, I posed the following strengthening of this as well in https://arxiv.org/abs/2601.04040 (Conjecture 18, $m = n$ case): is there a constant $\delta > 0$ such that for every tree decomposition of the $n \times n$ grid of width $n + a$, there is a vertex with spread at least $\delta(n/a)$, for all integers $a, n \geq 1$?

Since δ can be chosen and each vertex has spread at least 1, we may assume $a \leq n/1000$ for example. There is also always a path decomposition that achieves width $n + a$ and spread $2n/a + 2$ for each vertex. It is easier to prove a result such as conjectured above when the grid is unbalanced, say $m \times n$ for $m \geq n^2$.

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