Open Problems for the Barbados Graph Theory Workshop 2016

An up-to-date version of this list is maintained at `http://www.princeton.edu/~sspirkl/openproblems.html`.

Please send your open problems and comments to Sophie Spirkl (sspirkl@princeton.edu) to be included here.

The website for the workshop is `https://web.math.princeton.edu/~pds/barbados2016/index.html`.

1. Let G be a graph and D a tournament, with the same vertex set. For each vertex v, $A(v)$ denotes its outneighbour set in D. Suppose that for every vertex v, the chromatic number of $G[A(v)]$ is at most k. Is the chromatic number of G bounded by some function of k? This is true with chromatic number replaced by fractional chromatic number. *(Alex Scott and Paul Seymour)*

   Comments:

   1. The particular case of Borsuk graphs is very interesting and amounts to the existence of a tournament (or in fact semicomplete digraph) whose vertices are the points of the sphere S^d and out-neighborhoods have low chromatic number.

      For instance bipartite outneighborhoods would be that every out-neighborhood avoids a "fat equator" of the sphere. I think that the following question is extremely close: A *good function f* is a continuous function of S^d into the compact sets of S^d (here $f(x)$ is the outneighborhood of x) such that every $f(x)$ is disjoint from some equator of S^d.

      If there exists such an f which is semicomplete, i. e. where for all x, y we have x in $f(y)$ or y in $f(x)$, then we have a counterexample: this defines a semicomplete digraph with bipartite outneighborhoods (for some suitably chosen ε-Borsuk graph).

      But I think that good functions are never semicomplete, indeed if we now interpret $f(x)$ as the north pole of the avoided equator, we have the equivalent question: Is it true that for every continuous function f from S^d into itself there is x, y such that $f(x)$ is orthogonal to y, and $f(y)$ is orthogonal to x (call these pairs *xy bi-independent*)?

      I think this is true (apart when $d = 1$ where it suffices to take any small rotation) and doable. In fact, I think that the graph of bi-independent pairs is always of some large chromatic number (maybe d). If this is not the case, we can again form a counterexample I think. *(Stéphan Thomassé)*

   2. *In response to "Is it true that for every continuous function f from S^d into itself there is x, y such that $f(x)$ is orthogonal to y, and $f(y)$ is orthogonal to x (call these pairs xy bi-independent)?".*

      I think this much is true by an elementary argument. For each point x, let $S(x)$ be the points in S^d that are orthogonal to x (i. e. the copy of S^{d-1} around the equator). Then let $T(x) = S(f(x))$, so everything in $T(x)$ is orthogonal to $f(x)$. Now let $I(x)$ be the set of inner products

      $$\{\langle f(y), x \rangle : y \in T(x)\},$$

      so $I(x)$ is an interval, say $[m(x), M(x)]$. If 0 is in $I(x)$ we are done, as we can just pick a y that witnesses this. If 0 is never in $I(x)$ then, as $m(x)$ and $M(x)$ are continuous functions of x, we see that $I(x)$ is always the same sign. Let us show that this does not happen: pick x, x' antipodal and consider the intersection of $T(x)$ and $T(x')$. This must be nonempty (here is where we use $d > 1$) and so we can pick y in both. Then $\langle f(y), x \rangle$ and $\langle f(y), x' \rangle$ have opposite signs, but the first is in $I(x)$ and the second is in $I(x')$.

      Does that make sense? I would hope it to be possible to prove that the bi-independent pairs have large χ (for instance, there is room to pick several points and then use the fact that every set of $d - 1$ equators has a common point). *(Alex Scott)*

2. Suppose G is an Eulerian digraph which is strongly 100-edge-connected. Must there exist a partition of $E(G)$ into $\{A, B\}$ so that both $G \setminus A$ and $G \setminus B$ are strongly connected? *(Matt DeVos)*

3. If A, B are finite sets of integers and $|A + B| < |A| + 2|B| - 3$, must A be contained in an arithmetic progression of length at most $|A| + |B|$? *(Matt DeVos)*
   Update: False, but is it true that if $|A + B| < |A| + |B| + \sqrt{|A||B|} - 3$, and $|A| \le |B|$, then A is contained in an arithmetic progression of length $|A + B| - |A| + 1$? *(Matt DeVos)*

4. The *broadcast domination number* $\gamma_b(G)$ of a graph G is the minimum integer d such that there are pairs $(v_1, d_1), \ldots, (v_\ell, d_\ell)$, where v_i is a vertex of G and d_i is a positive integer, such that $d_1 + \cdots + d_\ell \le d$, and, for every vertex u of G, there is some index i with $\mathrm{dist}_G(u, v_i) \le d_i$. Surprisingly, the broadcast domination number can be computed in polynomial time for every graph [11]. Considering the dual of the LP relaxation of a natural IP formulation for broadcast domination, motivated the definition of so-called multipackings as dual objects (see [16] for a survey). A set M of vertices of G is a *multipacking* of G if for every vertex u of G and every integer d between 1 and the diameter of G, the set M contains at most d vertices at distance at most d from u. The duality of the corresponding LP relaxations implies that the *multipacking number $mp(G)$*, defined as the maximum cardinality of a multipacking in G, is a natural lower bound on the broadcast domination number $\gamma_b(G)$. It has been shown (cf. [16]) that the duality gap is 0 for trees and, more generally, strongly chordal graphs. Since the diameter is a trivial upper bound on $\gamma_b(G)$ and the set containing every third vertex of a longest shortest path is easily seen to be a multipacking, it follows that $\gamma_b(G) \le 3mp(G)$. *Can this trivial upper bound be improved?* Not even something like $3 - \epsilon$ seems to be known. *Can this bound be improved at least within certain graph classes?* A simple candidate class to look at first would be cacti. *(Dieter Rautenbach)*

5. The *burning number* $b(G)$ of a graph G [4] is the smallest integer k for which there are vertices $x_1, \ldots, x_k$ such that for every vertex u of G, there is some index i with $\mathrm{dist}_G(u, x_i) \le k - i$. Using the fact that $1 + 3 + 5 + \cdots + (2k - 1) = k^2$, it follows easily that $b(P_n) = \lceil \sqrt{n} \rceil$. Bonato et al. [4] conjecture that $b(G) \le \lceil \sqrt{n(G)} \rceil$ for every connected graph G. They prove $b(G) \le 2\sqrt{n(G)} + 1$, which we [3] managed to improve to $b(G) \le 1.3\sqrt{n} + 3$. Clearly, it suffices to show the conjecture for trees, which seems to indicate that it might be doable. Surprisingly though, the calculation of the burning number is already NP-hard for trees with exactly one vertex of degree at least 3 [3]. *(Dieter Rautenbach)*
   Update: Reformulation – given a tree of radius k^2, can it be covered with trees of radii $0, 1, \ldots, k$? *(Eli Berger)*

6. A graph is a $(q, q - 4)$-*graph* for some integer q at least 4 if every q vertices induce at most $q - 4$ distinct P_4s. Clearly, cographs coincide with $(4, 0)$-graphs. Quite surprisingly, $(q, q - 4)$-graphs have a very restricted structure [1], which allows to solve many problems efficiently for such graphs. Motivated by some problems on P_5-free graphs, we were recently led to consider graphs which might be called $(q, q - 5)$-*graphs* for some integer q at least 5, where every q vertices induce at most $q - 5$ distinct P_5s. Again, P_5-free graphs coincide with $(5, 0)$-graphs. *Do $(q, q - 5)$-graphs have a nice structure? Do at least triangle-free $(q, q - 5)$-graphs have a nice structure?* A triangle-free $(6, 1)$-graph for instance is either P_5-free or a subdivision of a star. *(Mitre Dourado, Lucia Penso, Dieter Rautenbach)*

7. Given a collection $\mathcal{N}$ of subsets of some finite ground set V, we consider the function

$$f : 2^V \to \mathbb{N}_0 : S \mapsto \min_{N \in \mathcal{N}} |N \setminus S|.$$

   For which collections $\mathcal{N}$ is this function submodular? If $\mathcal{N}$ consists of all k-element subsets of V for some integer k, then $f(S) = \min\{k - |S|, 0\}$, which is submodular. *Is this maybe*

(essentially) the only kind of collection for which f is submodular? We are interested in this question because submodularity of f would imply a logarithmic approximation guarantee for some simple greedy algorithm we considered. *(Carlos Vinícius Lima, Dieter Rautenbach, Uéverton Souza, Jayme Szwarcfiter)*

Update: Bases of matroids also satisfy this. *(Paul Seymour)*

Update: This characterizes set systems with this property, but the function is supermodular. *(Sang-il Oum)*

8. Let V be a set of order n. *For which vectors $(t_0, t_1, \ldots, t_n)$ does there exist a set system $\mathcal{T} \subseteq 2^V$ with*
$$\forall X \in \mathcal{T} : \forall Y \subseteq V : X \subseteq Y \Rightarrow Y \in \mathcal{T}$$
such that $t_i = \left| \mathcal{T} \cap \binom{V}{i} \right|$ for every i? Here $\binom{V}{i}$ is the set of all i-element subsets of V. *(Carlos Vinícius Lima, Dieter Rautenbach, Uéverton Souza, Jayme Szwarcfiter)*

Update: Kruskal-Katona theorem. *(Peter Keevash)*

9. Let $c, \varepsilon > 0$. We say that a graph G has (c, ε)-separators if every subgraph H of G has a balanced separator of order at most $c|V(H)|^{1-\varepsilon}$.

   Is there a way to "efficiently certify" that G has this property, at least in an approximate sense? Formally, do there exists $c', \varepsilon' > 0$ and a polynomial-time algorithm A such that

   (1) for each graph G with (c, ε)-separators, there exists a polynomial-size witness X such that A accepts (G, X), and

   (2) if G does not have (c', ε')-separators, then there does not exist any polynomial-size witness X such that A accepts (G, X).

   (Zdeněk Dvořák)

10. Conjecture (Brewster et al. [5]): *For $k \geq 3$, if $\chi(G) > k$, then G has at least $\frac{(k+1)(k-1)!}{2}$ cycles of length $0 \mod k$.*

    It follows from a lovely argument of Wrochna (unpublished; see [5]) that every graph G with $\chi(G) > k$ contains at least $\frac{(k-1)!}{2}$ cycles of length $0 \mod k$. If the above conjecture is true, then the complete graph on $k + 1$ vertices would be a tight example. The conjecture is open for every $k \geq 3$. *(Jon Noel)*

11. Let H be a fixed graph. A graph G on n vertices is said to be *weakly H-saturated* if the edges of $E(K_n) \setminus E(G)$ can be added to G, one edge at a time, so that every edge completes a copy of H when it is added. Let Q_d be the hypergraph of dimension d. *What is the minimum number of edges in a weakly Q_d-saturated graph on $n \geq 2^d$ vertices?* A related, but probably very different, problem was solved by Morrison, Noel and Scott [12]; many related problems and 'weak saturation' references can be found there. *(Jon Noel)*

12. Let G be an edge-colored d-regular bipartite graph where every color appears at most $\frac{d}{100}$ times. Does G have a perfect rainbow matching? (A perfect matching is rainbow if every color appears at most once in it). This is true if G is the complete bipartite graph [8]. *(Guillem Perarnau)*

13. *Conjecture:* For every graph G,
    (a) $\chi_f(G) \leq \text{had}(G)$ (b) $\chi(G) \leq \text{had}_f(G)$ (c) $\chi_f(G) \leq \text{had}_f(G)$.

    Definitions: Let $\chi(G)$ be the chromatic number of a graph G. The *Hadwiger number* $\text{had}(G)$ is the maximum t such that K_t is a minor of G.

    Let $\mathcal{S}(G)$ be the set of all stable sets in G. For $k \in \mathbb{Q}^+$, a *fractional k-colouring* of G is a function $q : \mathcal{S}(G) \to \mathbb{Q}^+$ such that

(i) $\sum(q(S) : S \in \mathcal{S}(G)$ and $v \in S) = 1$ for each vertex v, and

(ii) $\sum(q(S) : S \in \mathcal{S}(G)) \leq k$.

The *fractional chromatic number* $\chi_f(G)$ is the minimum $k \in \mathbb{Q}^+$ such that G is fractionally k-colourable.

Two subgraphs A and B of a graph G *touch* if $V(A) \cap V(B) \neq \emptyset$, or some edge of G has one endpoint in A and the other endpoint in B. A *bramble* in G is a set of connected subgraphs of G that pairwise touch. A set S of vertices in G is a *hitting set* of a bramble $\mathcal{B}$ if S intersects every element of $\mathcal{B}$. The *order* of $\mathcal{B}$ is the minimum size of a hitting set. The *bramble number* $\mathrm{bn}(G)$ is the maximum order of a bramble in G. Seymour and Thomas [15] proved that $\mathrm{bn}(G) = \mathrm{tw}(G) + 1$, where $\mathrm{tw}(G)$ is the treewidth of G.

The *fractional Hadwiger number* $\mathrm{had}_f(G)$ of G (introduced independently by Fox [9] and Pedersen [13]) is the maximum h for which there is a bramble $\mathcal{B}$ in G, and a function $w : \mathcal{B} \to \mathbb{R}_{\geq 0}$, such that $h = \sum_{X \in \mathcal{B}} w(X)$ and for each vertex v, the sum of the weights of the subgraphs in $\mathcal{B}$ that contain v is at most 1. *(David Wood)*

Comments:

1. The following inequalities are easily proved (see [10]):

$$\chi_f(G) \leq \chi(G) \quad \text{and} \quad \mathrm{had}(G) \leq \mathrm{had}_f(G) \leq \mathrm{bn}(G) = \mathrm{tw}(G) + 1.$$

Hadwiger's famous conjecture, $\chi(G) \leq \mathrm{had}(G)$, bridges the gap in the above inequalities. The above conjectures therefore are weaker than Hadwiger's conjecture. Note that Conjecture (a) implies Conjecture (c), and Conjecture (b) implies Conjecture (c). A greedy algorithm proves that $\chi(G) \leq \mathrm{tw}(G) + 1$.

So Conjecture (c) would seem a good starting point. Pedersen [13] presents the following equivalent formulation of Conjecture (c): for every graph G there is an integer t such that $\chi(G \cdot K_t) \leq \mathrm{had}(G \cdot K_t)$, where $G \cdot K_t$ is the lexicographic product obtained by replacing each vertex of G by K_t and each edge of G by $K_{t,t}$.

Note that Reed and Seymour [14] proved that $\chi_f(G) \leq 2\,\mathrm{had}(G)$. Conjecture (a) is due to Reed and Seymour [14]. Conjecture (b) is due to Harvey and Wood [10]. Conjecture (c) is independently due to Harvey and Wood [10] and Pedersen [13]. *(David Wood)*

14. A graph G is perfectly divisible if every induced subgraph H of G contains a set X of vertices such that X meets all largest cliques of H, and X induces a perfect graph. Question: does there exist an odd-hole-free graph that is not perfectly divisible? *(Chinh Hoang)*

15. Let G be a digraph. A "good colouring" means a partition of $V(G)$ such that for every vertex v, at most half of its outneighbours have the same colour as v. Odd cycles need three colours. Does every digraph in which every vertex has an out-neighbour have a good 3-colouring? *(Dominic van der Zypen and Paul Seymour)*
Update: Every such digraph has a good 4-coloring. *(Matt DeVos, David Wood)*

Comments:

1. What about colouring so that every vertex has a colour different from some outneighbour? Still, two colours is not always enough, but three is, and there's a pretty theorem – a strongly-connected digraph admits a two-colouring with this property if and only if the digraph has an even directed cycle. *(Paul Seymour)*

16. Let G, H be two finite, simple, undirected graphs. We call them "reduce-by-1"-isomorphic, or r_1-isomorphic for short, if there is a bijection $\psi : V(G) \to V(H)$ such that for all $v \in V(G)$

we have that $G \setminus \{v\}$ is isomorphic to $H \setminus \{\psi(v)\}$. (Ulam's reconstruction conjecture, or some version of it, states that r_1-isomorphic implies isomorphic.)

Question: Can we prove that if G, H are r_1-isomorphic then they have the same chromatic number, and the same Hadwiger number, matching number, intersection number, ...? *(Dominic van der Zypen)*

17. A graph H is a vertex-minor of a graph G if H can be obtained from G by applying a sequence of local complementations and vertex deletions, where a local complementation at a vertex v is an operation that flips the adjacency on every pair of neighbors of v.

    Let H be a graph. Does there exist a function f_H such that for every graph G with no H vertex-minor, $\chi(G) \leq f_H(\omega(G))$? Can you prove it for a wheel graph H? *(Jim Geelen, Sang-il Oum)*

    Comments:

    1. Dvořák and Král' [7] proved this when H is the wheel graph on 6 vertices. Choi, Kwon, and myself [6] proved this when H is a fan graph (wheel minus one vertex on the rim). *(Sang-il Oum)*

18. Let H be a graph. Does there exist a constant c depending only on H such that every n-vertex graph G with no induced subgraph isomorphic to a subdivision of H contains a clique or a stable set of size at least n^c? *(Sang-il Oum)*

    Comments:

    1. A weakening of Erdős-Hajnal conjecture. I wonder whether these forms have been studied. *(Sang-il Oum)*

19. Let G be a triangle-free graph, and let A, B be disjoint subsets of $V(G)$, such that A is stable and every vertex in B has (at least) a neighbour in A. Let $G[B]$ have huge chromatic number. Must there exist $X \subseteq A$ such that the set of vertices in B with distance two from X has large chromatic number? *(Alex Scott and Paul Seymour)*

20. Is there a polynomial time algorithm for the following problem:

    Input: A directed graph G and three pairs $(s_1, t_1), \ldots, (s_3, t_3)$ of vertices.
    Problem: Are there directed paths $P_1, \ldots, P_3$ such that P_i links s_i to t_i and no vertex of G is contained in more than 2 of the paths?

    Of course this problem can also be asked for $k > 3$ pairs of vertices but even for 3 it seems non-trivial. *(Stephan Kreutzer)*

21. We say that X is a dominating set of a tournament G if every vertex in $G \setminus X$ is an out-neighbor of some vertex in X. Is it true that for every tournament D, there exists a number k such that every tournament with no subtournament isomorphic to D has a dominating set with size at most k? *(Chun-Hung Liu)*

    Comments:

    1. The positive answer of the above problem implies a conjecture of Berger et al. [2] stating that for every k there exists c such that if G is a tournament in which the set of out-neighbors of each vertex has chromatic number at most k, then the chromatic number of G is at most c. This conjecture is is easy for $k = 1$, but open for all other k. This conjecture might also be related to the first problem in the list. *(Chun-Hung Liu)*

    2. Hehui Wu and I can prove the case when D can be obtained from the cyclic triangle by replacing each vertex by a transitive tournament. *(Chun-Hung Liu)*

3. The conjecture from Comment 1 also holds for $k = 2$. *(Stéphan Thomassé)*

22. The complexity of deciding whether a graph with no induced path on k vertices is 3-colorable is not known for $k \geq 8$. Here is a possibly easier question:

    Input: a graph G
    Output: G is not 3-colorable, or G is 100-colorable.

    Can this be done is polynomial time? What about other approximate versions? *(Maria Chudnovsky)*

    Update: For fixed k, this can be done because the class of graphs with no k-vertex path is χ-bounded. For $k = 8$, there is a polynomial-time algorithm that decides if G is 5-colorable, or not 3-colorable. *(Maria Chudnovsky and Maya Stein)*

23. Let $\pi \in S_n$ and $I \subseteq [n], I = \{i_1 < \cdots < i_k\}$, then let $\pi_I = (\pi_{i_1}, \ldots, \pi_{i_k})$. For I, J with $|I| = |J|$ we say $\pi_I \simeq \pi_J$ if the relative orderings of elements in π_I and π_J are the same. Define
    $$f(n) = \min_{\pi \in S_n} \max \left\{ k : \exists I, J \subseteq [n], |I| = |J| = k, I \cap J = 0, \pi_I = \pi_J \right\}.$$

    What is the asymptotic behavior of $f(n)$? It is known that $f(n) = \Omega(\sqrt{n})$ and $f(n) = O(n^{2/3})$. *(Guillem Perarnau)*

24. Does there exist a function f so that for every $c, \alpha > 0$ and every graph G, if for $X \subseteq V(G)$ is chosen uniformly at random, $G[X]$ is c-colorable with probability at least α, then $\chi(G) \leq f(c, \alpha)$? *(Zdeněk Dvořák)*
    Update: Yes. *(Sergey Norin)*
    Update: Simpler proof using A. Brace, D. E. Daykin. *A finite set covering theorem.* Bulletin of the Australian Mathematical Society 5.02 (1971): 197-202. *(Sang-il Oum, Stéphan Thomassé)*
    Update: f can be chosen as $3c + g(\alpha)$. *(Sergey Norin)*

25. Let $G = (A, B)$ be bipartite, $|A| = |B| = n$, with minimum degree $\delta(G) > (1/2 + \varepsilon)n$ and incidence matrix M so that $\det(M) \neq 0 \bmod 3$. Can we modify one entry of M so that its permanent (i. e. the number of perfect matchings between A and B) is not a multiple of 3? *(Stéphan Thomassé)*

26. Given $2^n + 1$ pairs (A_i, B_i) with $A_i \cap B_i = \emptyset$ and $A_i, B_i \subseteq [n]$, can you prove without using induction that $\exists i, j : |A_i \cap B_j| = 1$? *(Stéphan Thomassé)*

27. Let a, b be non-negative real numbers, and let G have average degree at least $a + b + 2$. Does there exist a partition $A, B \neq \emptyset$ of the vertices of G so that $G[A]$ has average degree at least a, and $G[B]$ has average degree at least b? *(Sergey Norin)*

28. A clique coloring of a graph is a coloring so that every maximal clique receives at least two colors (unless it is a single vertex). A unit disk graph is a graph whose vertices are represented by points in the plane, and two vertices are adjacent if and only if the corresponding points have distance less than 1. What is the minimum number k so that every unit disk graph has a clique coloring with at most k colors? It is known that $3 \leq k \leq 9$, where the lower bound follows from C_5. *(Colin McDiarmid)*

29. If G has $n \geq 7$ vertices and is an edge-maximal graph embeddable in a torus, does is follow that G has exactly $3n$ edges?

    More generally, call a class $\mathcal{C}$ of graphs *pure* if for each n, each n-vertex edge-maximal graph in $\mathcal{C}$ has the same number of edges. For which surfaces $\mathcal{S}$ is the class of graphs embeddable in $\mathcal{S}$ pure? *(Colin McDiarmid)*

30. For tournaments H_1, H_2, H_3, we denote a tournament T by $\Delta(H_1, H_2, H_3)$ if there is a partition (X_1, X_2, X_3) of $V(T)$ such that $T|X_i$ is isomorphic to H_i and X_i is complete to X_{i+1} for $i = 1, 2, 3$. $(X_4 = X_1)$ Let D_1 be a cyclic triangle and $D_n = \Delta(D_{n-1}, D_{n-1}, I)$ for $n \geq 2$ where I is a single vertex tournament. Let U_n be a tournament with $2n + 1$ vertices $\{v_1, v_2, \ldots, v_{2n+1}\}$ such that for $i < j$, v_j dominates v_i if and only if both i and j are odd. For $n \geq 2$, does there exist c such that every tournament excluding D_n and U_3 has chromatic number at most c? *(Ringi Kim)*

    Comments:

    1. If we exclude D_n and U_2, the answer is yes, and if we exclude D_n and U_4, the answer is no. *(Ringi Kim)*

31. A function $d : E(G) \to \mathbb{R}_{\geq 0}$ is called a *distance function* on edges of G if for all $uv \in E(G)$, the length of a shortest u-v-path in G with edges weights given by b is precisely $d(uv)$.

    An *embedding* of (G, d) in $\mathbb{R}^k$ is a set of points $q_v \in \mathbb{R}^k$ ($v \in V(G)$ s. t. for all $uv \in E(G)$, $\|q_u - q_v\|_{\ell_\infty} = d(uv)$.

    The parameter we are interested in is

    $$f(G) = \min\{k : (G, d) \text{ can be embedded in } \mathbb{R}^k \text{ for all distance functions } d \text{ on } G\}.$$

    It is known that $f(G)$ is a minor-closed parameter in the sense that if H is a minor of G, then $f(H) \leq f(G)$. The minimal excluded minors to ensure that $f(G) \leq k$ are known for $k = 1, 2$.

    Is $f(G)$ bounded when G is planar? If $f(G)$ bounded when G is a grid? If $f(G)$ bounded when G has bounded treewidth? *(Gwenael Joret)*

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