OPEN PROBLEMS

1. (Colin McDiarmid)

   A class of graphs $\mathcal{A}$ is *bridge-addable* if for every graph $G \in \mathcal{A}$ and every two components C, C' and all $x \in V(C)$ and $x' \in V(C')$, the graph obtained from G by adding the edge xx' belongs to $\mathcal{A}$.

   Conjecture: there is a constant $c > 0$ such that, for every bridge-addable class $\mathcal{A}$, if one draws uniformly at random an unlabelled graph G of size n from $\mathcal{A}$, then G is connected with probability at least c.

   Suppose that a graph is in the class $\mathcal{A}$ if and only if each component is; in this case we call $\mathcal{A}$ *decomposable*. If $\mathcal{A}$ is decomposable as well as bridge-addable, could it be true that the constant c above is at least the one for forests, for large n?

2. (Sergey Norin)

   There is a theorem of Haight asserting that for all k and l, there exists a digraph D with girth at least k and which is l-dominated (every subset of size l is dominated).

   Question: If D has size n and girth at least 4, what is the maximum l (in terms of n) for which D is l-dominated?

3. (Maria Chudnovsky)

   A *k-lift* G_k of a graph G is obtained by substituting a stable set S_x of size k for every vertex x of G, and then joining S_x with S_y by a perfect matching whenever xy is an edge of G.

   The number of perfect matchings $\mathrm{pm}(G_k)$ can be as large as $\mathrm{pm}(G)^k$, just by taking the disjoint union of k copies of G. This is not an upper bound when G is the triangle.

   Conjecture: If G is bipartite, then $\mathrm{pm}(G_k) \leq \mathrm{pm}(G)^k$.

4. (Zdenek Dvorak)

   When making the *full product* of G and H, if xy is an edge of G and uv is an edge of H, then all edges are added between $(x, u), (x, v), (y, u)$ and (y, v).

   The *full cube* of size n is the full product of three paths P_n.

   Conjecture. Suppose that S is a separator between the left face of the full cube and the right face; is it true that if n is large then the graph induced on S has large tree-width?

   SOLVED DURING WORKSHOP

5. (Sang-il Oum)

   There exists an algorithm which can test in time $f(k)n^3$ if the linear rankwidth of a graph is at most k; in other words, decides if there exists an enumeration of the vertices $v_1, v_2, \ldots, v_n$ in such a way that every partition $\{v_1, \ldots, v_i\}$ and $\{v_{i+1}, \ldots, v_n\}$ has bounded rank. (*Rank* of a partition X, Y means the rank over $GF(2)$ of the matrix indexed by $X \times Y$, where the entry for $u \in X$ and $v \in Y$ is 1 if u, v are adjacent and 0 otherwise.)

Question: Find such an algorithm (the existence proof is non-constructive).

6. (Dieter Rautenbach)

   If G is a d-regular graph with m edges, then every induced matching of G contains at most $m/(2d-1)$ edges.

   Can the graphs for which equality holds be recognized efficiently? These are exactly the d-regular graphs G whose vertex set can be partitioned into two sets X and Y such that X is independent and $G[Y]$ is 1-regular.

   If these graphs cannot be recognized efficiently, then one might consider approximating the maximum induced matching.

   The best known approximation algorithm for the maximum induced matching problem restricted to d-regular graphs is due to Gotthilf and Lewenstein and has a performance ratio of $0.75d + 0.15$.

   Can this be improved for d-regular graphs G with m edges that actually have an induced matching with (close to) $m/(2d-1)$ edges? That is, can one find a large induced matching in graph that are guaranteed to possess one?

7. (Stéphan Thomassé)

   The *triangle-free chromatic number* of a graph G is the maximum chromatic number of a triangle-free induced subgraph. Call this $\chi_3(G)$.

   Conjecture. There is a function f such that $\chi(G) \leq f(\chi_3(G), \omega(G))$.

8. (Paul Wollan)

   A class of hypergraphs H has the *Erdos-Posa Property* (EPP) if there is a function f for which the transversal number is bounded above by f(packing number).

   Problem: Find a natural class with EPP where the packing problem is not FPT.

9. (Paul Seymour)

   What is the complexity of the following problem?:

   Input: $s_1, t_1, s_2, t_2, s_3, t_3$ vertices of a digraph.

   Output: Find three s_i, t_i-paths such that no arc is used by all three of them.

10. (Sergey again)

    Question. Is there a triangle-free graph G such that for every subset X of size 4 (or even 1000) and for every stable set I in X, there is a vertex v with neighbourhood $N(v)$ say such that $N(v) \cap X = I$?