A Bound on χ for Graphs in $Forb^*(bull)$

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Abstract: Given a graph H, $Forb(H)$ is the class of all graphs that do not contain H as an induced subgraph, and $Forb^*(H)$ is the class of all graphs that do not contain any subdivision of H as an induced subgraph. A class $\mathcal{G}$ of graphs is *χ-bound* if there exists a function $f : \mathbb{N} \to \mathbb{N}$ (called a *χ-binding function for* $\mathcal{G}$) such that for all $G \in \mathcal{G}$, $\chi(G) \leq f(\omega(G))$. χ-bound classes of graphs were introduced in 1987 by András Gyárfás as a generalization of the class of perfect graphs. Gyárfás conjectured that for any tree T, the class $Forb(T)$ is χ-bound. In 1997, Alex Scott proved a 'topological' version of this conjecture: for any tree T, the class $Forb^*(T)$ is χ-bound; he then conjectured that for every graph H, the class $Forb^*(H)$ is χ-bound.

The *bull* is the graph consisting of a triangle and two disjoint pendant edges, and an *(m,n)-bull* is the graph obtained from the bull by subdividing one pendant edge $m-1$ times and the other $n-1$ times. In this talk, we present proofs of two special cases of Scott's conjecture: we first outline a proof of the fact that the class $Forb^*(bull)$ is χ-bound by a quadratic function, and then we discuss a proof that for all m and n, the class $Forb^*((m,n)-bull)$ is χ-bound by an exponential function.

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