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NICHOLAS M. KATZ

**Corrections to : “A conjecture in the arithmetic theory of differential equations”**

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Corrections to

**A CONJECTURE IN THE ARITHMETIC THEORY  
OF DIFFERENTIAL EQUATIONS (\*)**

BY

NICHOLAS M. KATZ

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I am indebted to O. GABBER for pointing out the following errors.

Section 10.6. — Delete the sentence “the Lie algebra  $\text{Lie}(G_{\text{gal}})$  is the smallest algebraic Lie sub-algebra of  $M(n)$  which contains the endomorphism

$$\begin{pmatrix} 2\pi i\lambda_1 & & & 0 \\ & \ddots & & \\ & & \ddots & \\ 0 & & & 2\pi i\lambda_n \end{pmatrix}''$$

It is both false (think of the case of all  $\lambda_i \in \mathbb{Z}$ ), and irrelevant to the correct calculation which follows.

Theorem 11.2 is incomplete in both hypotheses and proof as it is stated in the text. Here is a correct version.

**THEOREM 11.2.** — *Let  $(M, \nabla)$  be a rank two equation on an arbitrary  $X$ , whose determinant becomes trivial on a finite étale covering of  $X$ . Then we have  $\mathcal{G} = \text{Lie}(G_{\text{gal}})$  if any of the following conditions holds.*

- (A) *The Lie algebra  $\mathcal{G}$  contains non-nilpotent endomorphisms.*
- (B)  *$(M, \nabla)$  has non-nilpotent  $\psi_p$  for infinitely many  $p$ .*
- (C)  *$(M, \nabla)$  does not have regular singular points.*
- (D)  *$(M, \nabla)$  has regular singular points but it does not have rational exponents at infinity i. e., it does not have quasi-unipotent local monodromy at infinity.*

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(\*) Addendum à l'article de N. M. KATZ paru dans le fascicule II, tome 110, 1982, p. 203-239.

(E)  $\mathcal{G} \neq 0$ , and Grothendieck's conjecture holds for all rank one equations on all open subsets of  $X$ .

*Proof.* — Recall first that both (D), (C) imply (B), and (B) implies (A). If (A) holds, then the proof in the text is complete. If (A) does *not* hold, but (E) holds, then  $\mathcal{G}$  is the unipotent radical  $\mathcal{U}$  of a Borel. (This is the case overlooked in the text.) Then there exists a unique line  $L \subset M \otimes \mathbb{C}(X)$  which is  $\mathcal{G}$ -stable, and this line is killed by  $\mathcal{G}$ . As in case 2 in the text, this  $L$  must be horizontal. Then both  $L$  and  $M$  modulo  $L$  have nilpotent, hence zero,  $\psi_p$  for almost all  $p$ . Applying Grothendieck's conjecture to  $L$  and to  $M \bmod L$ , we find that  $\text{Lie}(G_{\text{gal}})$  lies in  $\mathcal{U}$ .

Q.E.D.

*Example 11.3.* — Replace “of infinite order” (true but irrelevant) by “not quasi-unipotent”.