Mathematics 104
Spring Term 2004
Final Examination
May 12, 2004

1. Evaluate $\int (\theta^2 + 1) \cos\theta \, d\theta$.

2. Evaluate $\int \frac{4xe^{x^2}}{e^{2x^2} + 2e^{x^2} + 2} \, dx$.

3. Evaluate $\int \frac{\sqrt{x^2 - 1}}{x^2} \, dx$. *Hint*: you may at some point want to use $\sin^2\theta = 1 - \cos^2\theta$.

4. Does $\int_0^\infty \frac{\sin^2 x}{x^2} \, dx$ converge or diverge? Give your reasons.

5. For each of the following three series, state whether it converges or diverges and give your reasons.

a) $\sum_{n=0}^{\infty} \frac{7^n - 2^n}{(2n)!}$.

b) $\sum_{n=1}^{\infty} \frac{n}{n^2 + \sqrt{n}}$.

c) $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{2^n + n^2}$.

6. For what values of x does each of the following two series converge? Give your reasons.

a) $\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{n^3}}$.

b) $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n}$.

7. Find the second order Taylor polynomial of $\tan^{-1} x$ about the center $a = \frac{1}{2}$.

8. Find $\sqrt[3]{1.01}$ with an error at most 0.0001. *Hint*: $\sqrt[3]{1.01} = (1 + 0.01)^{1/3}$.

9.

a) Draw the graph of the first two turns of the spiral given in polar coordinates by $r = 2\theta$ (that is, for $0 \le \theta \le 4\pi$).

b) Find the area of the region enclosed between the first and second turn of the spiral (i.e., the region between the curves $r = 2\theta$ for $0 \le \theta \le 2\pi$ and $r = 2\theta$ for $2\pi \le \theta \le 4\pi$, as well as the positive x-axis between 0 and 4π).

10. Find all real or complex solutions to $z^8 - z^4 - 2 = 0$.

11. The region inside the curve

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

and above the x-axis is revolved about the x-axis. Find the volume.

12. Solve the initial value problem

$$x\frac{dy}{dx} - 2y = x^3e^x, \quad y(1) = 0.$$