Mathematics 104
Fall Term 2000-2001
Final Examination
January 22, 2001

1. Evaluate $\displaystyle\int \frac{x^2}{x^6-1}\,dx$. Hint: try a substitution first.

2. Evaluate $\displaystyle\int e^{\sqrt{x}}\,dx$.

3. For each of the following integrals, state whether it is convergent or divergent and give your reasons.

a. $\displaystyle\int_1^\infty \frac{x^3\,dx}{\ln x + x^4}$.

b. $\displaystyle\int_0^\infty \frac{dx}{x^3+\sqrt{x}}$.

4. Find each of the following limits or show that it does not exist.

a. $\displaystyle\lim_{n\to\infty} \frac{n+17\tan^{-1} n+2}{1-n}$.

b. $\displaystyle\lim_{n\to\infty} n^2\left(1-\cos\frac{1}{n}\right)$.

5. For what x does the following series converge? Give your reasons.

$$\sum_{n=2}^{\infty} \frac{(2x-1)^n}{n\ln n}.$$

6. Find the Taylor series at 0 of

$$f(x)=\frac{1-\cos(2x^2)}{x}$$

and find $f^{(7)}(0)$ and $f^{(8)}(0)$.

7. Find all complex numbers z, in Cartesian (rectangular) form, such that

$$(z-1)^4=-1.$$

8. For what θ does

$$\sum_{n=0}^{\infty} \frac{\cos n\theta}{2^n}$$

converge? Evaluate the series.

9. Find all real solutions of the following differential equations:

a. $y'' + 4y' + 13y = 0$.

b. $y'' + 4y' + 13y = 13x^2 - 5x + 24$.

10. Find the arc length of the curve given by

$$y = \frac{x^2}{2} - \frac{\ln x}{4}$$

for x in the interval $[2, 3]$. Hint: the quantity under the square root sign can be rewritten as a square.

11. Let R be the region bounded by $y = x + x^2$, $x = 1$, $x = 2$, and the x-axis. Consider the solid formed by revolving R about

a. the y-axis

b. the line $x = 3$

c. the x-axis.

In each case express the volume of the solid as a definite integral, but *do not evaluate the integral.*