Mathematics 104

Practice Quiz 3C

1. Let $I = \displaystyle\int \frac{1}{1+e^x}\,dx$. Substitute for the troublesome part of the integrand: let $y = e^x$. Then $x = \ln y$ and $dx = dy/y$, so $I = \displaystyle\int \frac{1}{1+y}\frac{dy}{y}$. Set

$$\frac{1}{(1+y)y} = \frac{A}{1+y} + \frac{B}{y}, \quad 1 = Ay + B(1+y).$$

For $y = -1$ we find $A = -1$ and for $y = 0$ we find $B = 1$. Hence

$$I = -\ln|1+y| + \ln|y| + C = -\ln(1+e^x) + \ln e^x + C = -\ln(1+e^x) + x + C.$$

Notice that since $y = e^x$, y and $1+y$ are automatically positive, and we can drop the absolute value signs.

2.

$$\text{Let } I = \int x \ln x\,dx,$$

$$u = \ln x, \qquad dv = x\,dx,$$

$$du = \frac{dx}{x}, \qquad v = \frac{1}{2}x^2.$$

Then

$$I = \frac{1}{2}x^2\ln x - \frac{1}{2}\int \frac{x^2}{x}\,dx = \frac{1}{2}x^2 \ln x - x^2 + C.$$

(Thanks to Dale Shepherd for catching the numerical mistake in the previous answer.)

3. Let $I = \displaystyle\int_{1/\sqrt{2}}^{1} \sqrt{1-x^2}\,dx$ and $x = \sin\theta$. Then $\sqrt{1-x^2} = \cos\theta$ and $dx = \cos\theta\,d\theta$. When $x = 1$, $\theta = \pi/2$ and when $x = 1/\sqrt{2}$, $\theta = \pi/4$. Therefore

$$\begin{aligned} I &= \int_{\pi/4}^{\pi/2} \cos^2\theta\,d\theta = \int_{\pi/4}^{\pi/2} \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta \\ &= \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\Big|_{\pi/4}^{\pi/2} = \frac{\pi}{4} - \left(\frac{\pi}{8} + \frac{1}{4}\right) = \frac{\pi}{8} - \frac{1}{4}. \end{aligned}$$

Note: expressions like $\sin^{-1} 1$ *and* $\sin^{-1}\frac{1}{\sqrt{2}}$ *should be simplified.*