Mat104 Fall 2002, Infinite Series Problems From Old Exams

For the following series, state whether they are convergent or divergent, and give your reasons.

(1) $\sim \frac{1}{n}$, diverges by the limit comparison test (LCT)
(2) converges by ratio test
(3) converges by ratio test
(4) $\sim \frac{1}{n}$, diverges by LCT
(5) $\sim \frac{2^n}{3^n}$, converges by LCT
(6) converges by the alternating series test (AST). It is conditionally convergent only since taking absolute values gives a divergent sum. ($\ln n < n$ implies that $\ln(\ln n) < \ln n$ so $\frac{1}{\ln(\ln n)} > \frac{1}{\ln n} > \frac{1}{n}$)
(7) converges by AST, conditionally convergent since summing $1/\sqrt{n}$ gives a divergent series (p-test with p = 1/2).
(8) convergent by LCT
(9) $\sim \frac{1}{n^3}$ so convergent by LCT
(10) diverges since $a_n \to \infty$ as $n \to \infty$.
(11) $\sim \frac{1}{n}$ so divergent by LCT
(12) convergent by the ratio test
(13) conditionally convergent
(14) converges by ratio test. $a_{n+1}/a_n \to 1/e$.
(15) converges by the nth root test.
(16) convergent by LCT. Asymptotic to $\frac{2^n + 6^n}{7^n}$, the sum of two convergent geometric series.
(17) divergent since $a_n \to e$.
(18) divergent by LCT since $\sim \frac{1}{\sqrt{n}}$
(19) convergent by LCT since $\sim \frac{1}{n^{3/2}}$
(20) convergent by the ratio test
(21) convergent by AST. Conditionally convergent only since $\frac{1}{\ln^2 n + 2} \sim \frac{1}{\ln^2}$ and $\frac{1}{\ln^2 n} > \frac{1}{n \ln n}$ which gives a divergent sum by the integral test. ($\ln(\ln x)) \to \infty$ as $x \to \infty$.
(22) converges by the ratio test.
(23) divergent since $\frac{\ln n}{n} > \frac{1}{n}$, which diverges by the p-test.
(24) $\sim \frac{1}{n^2}$ so converges
(25) converges by the integral test. (Make the substitution $u = \ln(\ln x)$.
(26) $\sim \frac{5}{2n^2}$ so converges.
(27) conditionally convergent.
(28) sum of geometric series with $r = 1/2$ and $r = -1/6$.
(29) convergent by the ratio test
(30) convergent by the ratio test

(31) $\sim \dfrac{1}{n^2}$ so converges.
(32) difference of convergent geometric series
(33) $\sim \dfrac{1}{n}$ so diverges.
(34) divergent geometric series with $r > 1$.
(35) convergent since $\leq \dfrac{1}{n^2+1}$
(36) divergent. 7^n dominates. Divide top and bottom by 7^n and take the limit
(37) converges, behaves like $\left(\dfrac{5}{7}\right)^n - \left(\dfrac{2}{7}\right)^n$, difference of two convergent geometric series
(38) $\sim \dfrac{1}{n}$ so diverges.
(39) converges by the ratio test.
(40) $\ln(n^2+1) \sim \ln(n^2) = 2\ln n$. So $n\ln(n^2+1) \sim n\ln n$ and this diverges by the integral test. So both diverge.
(41) converges by the ratio test
(42) absolutely convergent. bounded by $\dfrac{1}{n^2\ln n}$ which converges by comparison to $1/n^2$.
(43) diverges by the integral test. (Take the derivative of $\ln(\ln(\ln x))$).
(44) $\sim \dfrac{1}{n}$ so diverges
(45) conditionally convergent
(46) divergent since $a_n \to \pi/2$
(47) divergent since $a_n \to e^2$
(48) converges by the root test
(49) $\sim \dfrac{n}{n^2}$ so diverges
(50) converges by the ratio test – Use L'Hôpital's rule to show that $\dfrac{\ln(n^2+2n+2)}{\ln(n^2+1)} \to 1$.
(51) converges by the ratio test
(52) difference of convergent geometric series
(53) compare to $\dfrac{1}{n\ln^2 n}$. By the integral test this series converges, so both converge.
(54) behaves like $\dfrac{1/n}{\sqrt{n}}$ since $\sin(1/n) \sim 1/n$ when n is large and $\cos(1/n) \approx 0$ when n is large.
(55) $\sim \dfrac{e}{n^2+1}$ so converges.
(56) converges by the ratio test (or the root test if you know that $n^{1/n}$ goes to 1 as n goes to infinity.)
(57) converges by the ratio test