Mat104 Fall 2002, Integration Problems From Old Exams

Warning: Many of these integrals can be done several different ways. If you choose a different method than I did, your answer may look quite different from the answer given here. The two different-looking answers may simply differ by a constant or perhaps they can be seen to be the same through the clever use of identities. If you believe that your answer is correct, but it does not match the one given here, consult your instructor! If you find errors, please let me know (jmjohnso@math.princeton.edu).

(1) $\cos^2 x - \dfrac{\cos^4 x}{4} + \ln|\sec x| + C$

(2) $\dfrac{1}{16}\dfrac{x}{\sqrt{4+x^2}} - \dfrac{1}{48}\cdot\dfrac{x^3}{(\sqrt{4+x^2})^3} + C$

(3) $-2\sqrt{1+x}\,\cos\sqrt{1+x} + 2\sin\sqrt{1+x} + C$

(4) $x\arctan x - \ln\sqrt{1+x^2} + C$

(5) $\dfrac{3x}{8} + \dfrac{\sin 2x}{4} + \dfrac{\sin 4x}{32} + C$ or, equivalently, $\dfrac{3x}{8} + \dfrac{\cos^3 x\sin x}{4} + \dfrac{3}{8}\cos x\sin x + C$

(6) $\dfrac{1}{4}\ln 3$

(7) $\ln(1+\ln x) + (\ln x)\ln(1+\ln x) - \ln x + C$

(8) $\dfrac{x^3\arctan x}{3} - \dfrac{x^2}{6} + \dfrac{\ln\sqrt{1+x^2}}{3} + C$

(9) $\dfrac{1}{8\sqrt{3}}$

(10) $\dfrac{e^{x^2}}{4}(\sin x^2 - \cos x^2) + C$

(11) $\ln|x + \sqrt{x^2+25}| + C$

(12) $u^3 - 3u + \dfrac{3}{4}\ln|u-1| + \dfrac{21}{8}\ln|u^2+u+2| + \dfrac{39}{4\sqrt{7}}\arctan\left(\dfrac{2u+1}{\sqrt{7}}\right) + C$ where $u = \sqrt[3]{x+2}$

(13) $3x - 4\ln|x+2| + \ln|x-1| + C$

(14) $3\sqrt[3]{x}\sin\sqrt[3]{x} + 3\cos\sqrt[3]{x} + C$

(15) Assume that $x > 0$ for simplicity. In that case, the answer is $\ln|x+1+\sqrt{x^2+2x}| + C$

(16) $\ln(x^2+6x+10) + \arctan(x+3) - \ln|x+1| + C$

(17) $-\ln\left|\dfrac{1}{x} + \dfrac{\sqrt{1-x^2}}{x}\right| + C$ (if we use $-\ln|\csc\theta + \cot\theta|$ as our antiderivative for $\csc\theta$.) Alternatively, we might use $\ln|\csc\theta - \cot\theta|$ as an antiderivative for $\csc\theta$ and this would give $\ln\left|\dfrac{1}{x} - \dfrac{\sqrt{1-x^2}}{x}\right| + C$ instead.

(18) $\frac{1}{2}(x^2e^{x^2} - e^{x^2}) + C.$

(19) $\frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$

(20) $\frac{(\sqrt{1-x^2})^3}{3} - \sqrt{1-x^2} + C$

(21) $\frac{\tan^3 \theta}{3} - \tan\theta + \theta + C$

(22) $\ln\sqrt{x^2+4x+13} - \frac{\arctan((x+2)/3)}{3} + C$

(23) $\ln(9/8)$

(24) $2\arctan(e^{x/2}) + C$

(25) $\ln|x| + \ln\sqrt{x^2+2x+10} + \frac{2}{3}\arctan\left(\frac{x+1}{3}\right) + C$

(26) $-\frac{(\sqrt{9-x^2})^3}{3} - 9\arcsin\left(\frac{x}{3}\right) - x\sqrt{9-x^2} + C$

(27) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

(28) $1 - \frac{2}{e}$

(29) $x(\ln x)^2 - 2x\ln x + 2x + C$

(30) $-2\sqrt{1+\cos x} + C$

(31) $\frac{1}{6}\ln\left|\frac{x^3-1}{x^3+1}\right| + C$

(32) $\frac{2}{5}\cos^5 x - \frac{\cos^7 x}{7} - \frac{\cos^3 x}{3} + C$

(33) $x - 2\ln|1-e^x| + C$

(34) $\frac{e}{2}(\sin 1 - \cos 1) + \frac{1}{2}$

(35) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$

(36) $\frac{x}{4\sqrt{4-x^2}} + C$

(37) $\sqrt{e} - \sqrt[3]{e}$

(38) $\frac{\operatorname{arcsec}(x/2)}{4} - \frac{\sqrt{x^2-4}}{2x^2} + C$

(39) $-\ln|x| + \ln\sqrt{x^2+1} + \arctan x + C$

(40) $-\frac{1}{4} \cdot \frac{\sqrt{x^2+4}}{x} + C$

(41) $-2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

(42) $\ln|x| - \ln|x-1| - \frac{1}{x-1} + C$

(43) $\frac{(\sqrt{x-1})^4}{2} + \frac{2(\sqrt{x-1})^3}{3} - 6(\sqrt{x-1})^2 + C$

which can be simplified to $\frac{(x-1)^2}{2} + \frac{2}{3}(x-1)^{3/2} - 6(x-1) + C$

or, simplifying further, $\frac{x^2}{2} - 7x + \frac{2}{3}(x-1)^{3/2} + C$.

(44) $(x^2+3x)\ln x - \frac{x^2}{2} - 3x + C$

(45) $-\frac{\sqrt{9+x^2}}{x} + \ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right| + C$

(46) $\frac{1}{4}\ln(x^2+1) + \frac{1}{2}\arctan x - \frac{1}{2}\ln|x+1| + C$

(47) $\frac{(1+e)^{21}}{21} - \frac{2^{21}}{21}$

(48) $x - 2\ln(x^2+4x+5) + 3\arctan(x+2) + C$

(49) $\frac{1}{2}\ln(x^2+2x+3) + C$