Mat104 Solutions to Problems from Old Exams
Geometric Series, Sequences and L'Hôpital's Rule

(1) Since $e^{nx} = (e^x)^n$ this is a geometric series with $r = e^x$. It converges absolutely, provided $|e^x| < 1$, that is for $x \in (-\infty, 0)$. In that case, it will converge to $\dfrac{1}{1-e^x}$.

(2) This is a geometric series with $a = (-2/3)^4$ and $r = -2/3$. Therefore it converges to $\dfrac{(-2/3)^4}{1+2/3} = \dfrac{16}{135}$.

(3) Here we combine several geometric series:

$$\sum_{n=0}^{\infty} \frac{2^n}{5^n} = \frac{1}{1-2/5} = 5/3 \quad \sum_{n=0}^{\infty} \frac{3^{n+1}}{5^n} = \frac{3}{1-3/5} = 15/2 \quad \sum_{n=0}^{\infty} \frac{4^{n+2}}{5^n} = \frac{16}{1-4/5} = 80$$

the series we are given will converge to $5/3 + 15/2 + 80 = \dots$.

(4) Answer: $2 + 1/2 - 3/8 = 17/8$ (similar to problems 1-3 above)

(5) Answer: $8/3 + 2 = 14/3$. (similar to problems 1-3 above)

(6) As $n \to \infty$, both the numerator and the denominator go to infinity. Thus we can use L'Hôpital's Rule:

$$\lim_{n\to\infty} \frac{\ln(n^2+n)}{\ln(n^2-n)} = \lim_{n\to\infty} \frac{\dfrac{2n+1}{n^2+n}}{\dfrac{2n-1}{n^2-n}} = \lim_{n\to\infty} \frac{2n+1}{2n-1} \cdot \frac{n^2-n}{n^2+n} = 1$$

since the leading term on top and bottom is now $2n^3$.

(7) This limit will be 0. If we apply L'Hôpital's Rule repeatedly we end up with $\dfrac{18}{24n}$ which goes to 0 as n goes to infinity.

(8) This limit exists and equals -1. (Recall $\arctan n$ goes to $\pi/2$ as n goes to infinity.)

(9) Since $(1 + 1/n)^n$ goes to e as n goes to infinity, this will go to e^2.

(10) (a) The numerator goes to e and the denominator goes to ∞. So the quotient will go to 0 as n goes to ∞.

(b) An $\dfrac{\infty}{\infty}$ form so use L'Hôpital's Rule to show that the limit is 1/2.