Mat104 Fall 2002, Problems on Area, Volume and Length From Old Exams

(1) (a) The region S bounded by $y = \sec x$ and the x-axis for $-\pi/4 \le x \le \pi/4$ is rotated around the x-axis. Find the volume of the resulting solid.

(b) The region R bounded by the parabolas $y^2 = x$ and $y^2 = 2x - 6$ is rotated about the x-axis. Find the volume of the resulting solid.

(2) Find the arc length of the of the curve given by $x = e^{2t}\sin 2t$, $y = e^{2t}\cos 2t$ for $0 \le t \le 1$.

(3) The region enclosed between the curve $y = e^{-x}$ and the lines $x = 1$ and $x = 2$ is rotated around the x-axis. Find the volume of the resulting solid of revolution and the surface area of the boundary of this solid.

(4) Let R be the region above the x-axis, to the right of the y-axis, and below the circle of radius 1 and center $(1, 1)$. Find the area of R. Find the volume of the solid S obtained by rotating the region R around the x-axis. Find the surface area of this solid S.

(5) Let R be the region bounded by the lines $y = 0$, $y = 1$, $x = 2 - y$, and the curve $x = \sqrt{y}$.
(a) Find the area of the region R.
(b) Find the volume when the region R is revolved about the line $y = 1$.
(c) Find the volume when the region R is revolved about the y-axis.

(6) Let γ be the parametrized curve given by

$$(x, y) = (1 - \sin t, 2 + \cos t), \qquad \text{where } t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

(a) Find the equation of the line tangent to the curve γ at the point $(1, 3)$.
(b) Find the surface area produced when the curve γ is revolved about the line $y = 2$.

(7) The region above the curve $y = x^2 - 6x + 8$ and below the x-axis is revolved around the line $x = 1$. Find the volume of the resulting solid.

(8) Find the arc length of the curve given by

$$y = \frac{x^2}{2} - \frac{\ln x}{4}$$

for x in the interval $[2, 3]$. (Hint: the quantity under the square root can be rewritten as a perfect square.)

(9) Let R be the region bounded by $y = x + x^2$, $x = 1$, $x = 2$, and the x-axis. Consider the solid formed by revolving R about
(a) the y-axis
(b) the line $x = 3$
(c) the x-axis

In each case express the volume of the solid as a definite integral, but **do not evaluate the integrals.**

(10) Consider the curve given in parametric equations by

$$x = t\cos t \qquad y = t\sin t$$

Find a formula for the slope dy/dx of the curve at the point corresponding to t, and find the tangent line to the curve at the point $t = \pi/2$.

(11) Consider the region R under the curve $y = e^{x^2}$ and above the interval $0 \le x \le 1$. Find the volume of the solid obtained by revolving the region R around the y-axis. Also, express as a definite integral the volume of the solid obtained by revolving the region R around the x-axis (but do not attempt to evaluate this integral).

(12) Consider the curve given in parametric equations by

$$x = e^t \qquad y = e^{2t} + 1 \quad \text{for } 0 \le t \le 1$$

Find the surface area of the surface obtained by rotating this curve around the y-axis.

(13) The region between $y = x^{1/3}$, the x-axis, and the line $x = 1$ is revolved around (a) the x-axis, (b) the y-axis. Find the volume in each case.

(14) Find the arc length of

$$y = \cosh x \qquad \left(\text{in other words, of } y = \frac{e^x + e^{-x}}{2}\right)$$

as x runs from 0 to 1.

(15) Consider the curve given by the parametric equations

$$x = t \qquad y = t^2 \qquad \text{for } 0 \le t \le 1.$$

(a) Compute the volume of the solid obtained by revolving the region bounded by the curve, the y-axis, and $y = 1$ around the y-axis.
(b) Compute the area of the surface of this solid.

(16) The region under the arch of the cycloid

$$x = a\theta - a\sin\theta \qquad y = a - a\cos\theta \qquad 0 \le \theta \le 2\pi$$

is revolved around the x-axis. Find the volume of the solid of revolution produced.

(17) Find the area of the surface generated by revolving the parabolic arc $y = x^2$ for $0 \le x \le 1$ about the y-axis.