MATH 104 - FINAL EXAM
Wednesday, May 14, 2003, 1:30PM-4:30PM
McCosh 50

1. *(10 points)* Find the following integrals.

   (a) $\int e^{2x} \sin(e^x)\, dx$

   (b) $\int_0^1 \frac{x^2}{(\sqrt{4-x^2})^3}\, dx$

2. *(12 points)*

   (a) Let R be the region bounded by the curve $y = x^3$, the x-axis and the two vertical lines $x = 1$ and $x = 2$. Find the volume of the region obtained by rotating R about the line $x = 3$.

   (b) Let C be the portion of the curve $y = x^3$ between the points $(1, 1)$ and $(2, 8)$. Find the area of the surface generated by rotating C about the x-axis.

3. *(15 points)* Determine whether the given improper integrals converge or diverge. Justify your answers.

   (a) $\int_2^\infty \frac{\sin^2(x)}{x(\ln x)^2}\, dx$

   (b) $\int_0^1 \frac{\sin(x^2)}{x^{5/2}}\, dx$

   (c) $\int_1^\infty \frac{\arctan(x^2)}{x^3 + \sqrt{x}}\, dx$ Note: $\arctan(x^2) = \tan^{-1}(x^2)$

   (d) $\int_0^1 \frac{x^{3/2}}{\ln(1+x^2)}\, dx$

   (e) $\int_1^\infty \frac{dx}{x^2-1}$.

4. *(15 points)* Write **AC** or **CC** or **D** to indicate whether the given series is **A**bsolutely **C**onvergent, or **C**onditionally **C**onvergent or **D**ivergent. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n} + (-5)^n}{5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, 2^n}{\sqrt[3]{n!}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sin(1/n)(\sqrt[n]{e} - 1)}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\left(1 + \frac{1}{\sqrt{n}}\right)^n}{n^{n/2}}$

5. *(10 points)* Let $0 \leq \theta \leq 2\pi$ and consider the series $\sum_{n=1}^{\infty}(-1)^{n+1}\tan^{2n}(\theta)$. Determine the values of θ for which the series converges and compute the sum. Simplify your answer.

6. *(9 points)* Find the first three nonzero terms of the Taylor series at 0 for the function $f(x) = \dfrac{\sin x}{1+x^3}$.

7. *(10 points)* Find $\displaystyle\lim_{x\to 0}\frac{(e^{2x^2}-1-2x^2)(\cos(x)-1)}{[\sin(3x)-\ln(1+3x)]x^4}$

8. *(9 points)* Let $z = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i$ and let w be the complex number whose modulus is 2 and whose argument is $\pi/3$. (**Note:** The modulus of a complex number is the same as the magnitude.) Write each of the quantities below in the form $a + ib$ where a and b are real numbers.

   (a) $\dfrac{1}{z}$

   (b) z^{80}

   (c) $z^2 \cdot w$

9. *(10 points)* Find all complex numbers z satisfying the equation $(2z-1)^4 = -16$. Express your answers in the form $a + ib$, where a and b are real.