Erratum:
A Note on Competitive Diffusion Through Social Networks

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As noted by Takehara et al. [1], Theorem 2.1 in our paper is incorrect. In particular, the statement "If the diameter of the graph is two then vertices can only be colored by an agent $i \in N$ at time 1 or 2", which appears in the theorem's proof, is false. Takehara et al. in fact provide an explicit counterexample to the theorem.

The following is a correct statement of the theorem.

Theorem 2.1. Let G be a graph and let $N = \{1, \ldots, n\}$ such that $D(G') \leq 2$ for every G' that is obtained from G by removing $n-1$ vertices along with their neighbors. Then the game $\Gamma = \langle G, N \rangle$ admits a Nash equilibrium, which can be found in polynomial time.

Alternatively, the theorem is correct as stated if the model is modified to allow gray vertices to infect their neighbors, that is, at time $t+1$ any white vertex that has a gray neighbor becomes gray.

Note that random graphs on m vertices $G(m,p)$ satisfy the above property with high probability for $p = \Omega\left(\sqrt{(n \ln m)/m}\right)$. Indeed, given two vertices $u, v \in V$, when one deletes $n-1$ other vertices, the probability that there is no $w \in V$ that is adjacent to u and v but not to the deleted vertices is

$$\left[1 - p^2(1-p)^{n-1}\right]^{m-(n+1)}.$$

When $pn = o(1)$, taking $p \geq \sqrt{(4n \ln m)/m}$, this expression is smaller than

$$\left(1 - \frac{2n \ln m}{m}\right)^m \leq \left(\frac{1}{e}\right)^{2n \ln m} = \frac{1}{m^{2n}}.$$

The probability is still small after applying the union bound to all $\binom{m}{2} \cdot \binom{m-2}{n-1} < m^{n+1}$ choices of a pair of vertices and $n-1$ others.

References

[1] R. Takehara, M. Hachimori, and M. Shigeno. A comment on pure-strategy Nash equilibria in competitive diffusion games. Manuscript, 2011.

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