Curvature-based Analysis of Connectivity Structure in Brain Networks

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Abstract

Brain networks inferred from collective patterns of neuronal activity are cornerstones of experimental neuroscience. Modern fMRI scanners allow for high-resolution data that measures the neuronal activity underlying cognitive processes in unprecedented detail. Due to the immense size and complexity of such data sets, efficient evaluation and visualization remain data analysys challenges.

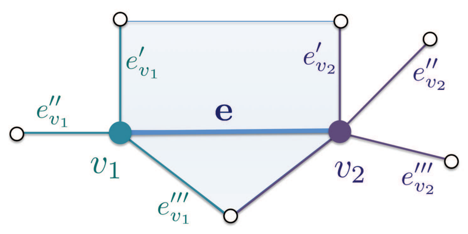
In this study, we combine recent advances in experimental neuroscience and applied mathematics to perform a mathematical characterization of complex networks constructed from fMRI data. We use task-related edge densities (G. Lohmann et al., PlosOne 2016) for constructing networks whose nodes represent voxels in the fMRI data and edges the task-related changes in synchronization between them. This construction captures the dynamic formation of patterns of neuronal activity and therefore represents effectively the connectivity structure between brain regions.

Using geometric methods that utilize Forman-Ricci curvature as an edge-based network characteristic (M. Weber et al., J Complex Networks 2017), we perform a mathematical analysis of the resulting complex networks. We motivate the use of edge-based characteristics to evaluate the network structure with geometric methods. Our results identify unique features in the network structure including long-range connections of high curvature acting as bridges between major network components.

Curvature-based Analysis

Classic network analysis has focused on the elements of the system and their connectivity (node-based approach) rather than the relations (edges) between them.

We propose an edge-based approach:
- evaluate not only binary, but also weighted networks;
- natural notion for directed networks;
- dynamic models for network evolution;
- generalization from pairwise to higher order interactions.



Forman-Ricci Curvature

For a network $G=\{V,E\}$ with edge weights $\omega(e)$ and node weights $\omega(v)$ we define

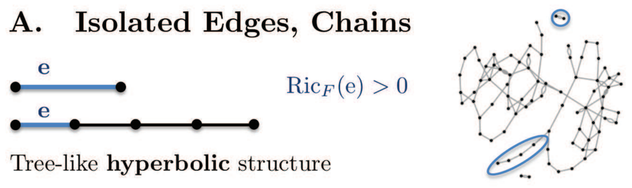
$$\mathrm{Ric}_F(e)=\omega(e)\left(\frac{\omega(v_1)}{\omega(e)}+\frac{\omega(v_2)}{\omega(e)}\right)-\sum_{\substack{e_{v_1}\sim e\\ e_{v_2}\sim e}}\left(\frac{\omega(v_1)}{\sqrt{\omega(e)\omega(e_{v_1})}}+\frac{\omega(v_2)}{\sqrt{\omega(e)\omega(e_{v_2})}}\right)$$

Our approach builds on a discrete version of the well-known concept of *curvature* in differential geometry. The edge-based Forman curvature and its associated geometric flow can be utilized to
- identify higher order connectivity structure in complex networks;
- characterize local assortativity;
- detect structural anomalies.

▼ geometric motivation

Binary Networks: $\mathrm{Ric}_F(e)=4-\sum_{v\sim e}\deg(v)$

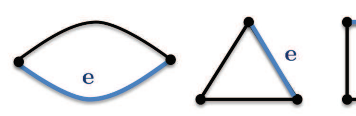
A. Isolated Edges, Chains



$\mathrm{Ric}_F(e)>0$

Tree-like **hyperbolic** structure

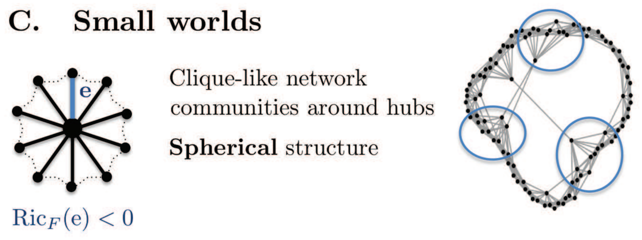
B. Cycles



$\mathrm{Ric}_F(e)=0$

Curvature vanishes on cycles
Euclidean structure

C. Small worlds



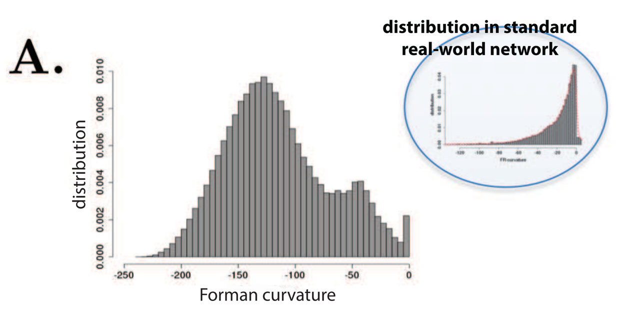
Clique-like network communities around hubs
Spherical structure

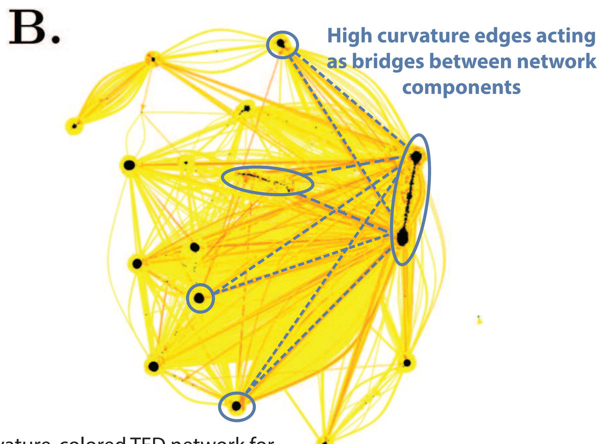
$\mathrm{Ric}_F(e)<0$

M. Weber et al. J. Complex Net. '17

Results

1. Geometric Analysis

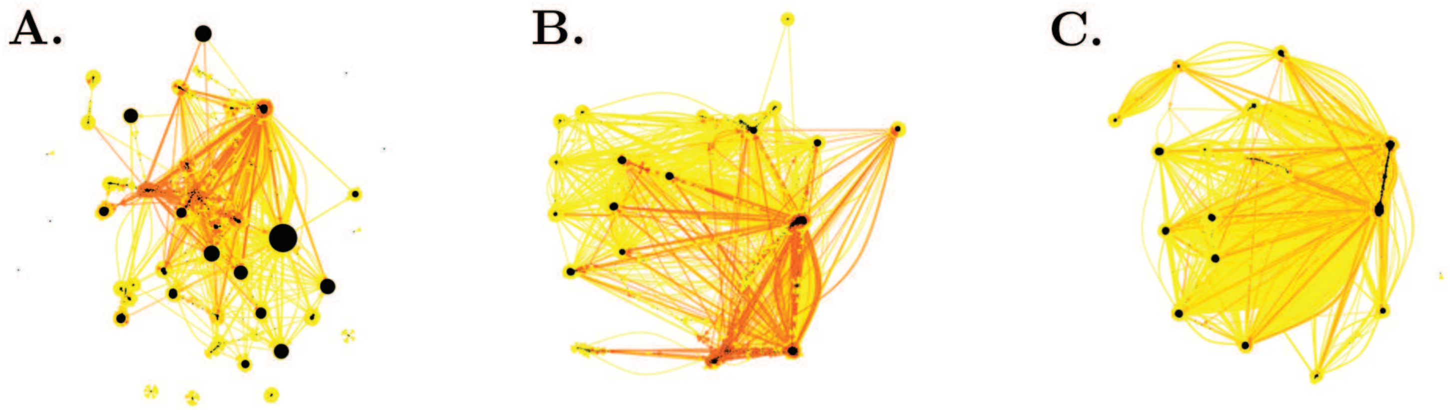




Curvature-colored TED network for motor task (source: Human Connectome project).

- Globally negative curvature indicating community structure (A).
- Distribution deviates from observations in other real-world networks (A): Broad-scale distribution instead of power-law distribution.
- Distribution shows a secondary peak as we observed in different types of correlation-based networks (A).
- Major network communities are connected by high curvature edges (B).

2. Higher Order Network Organization

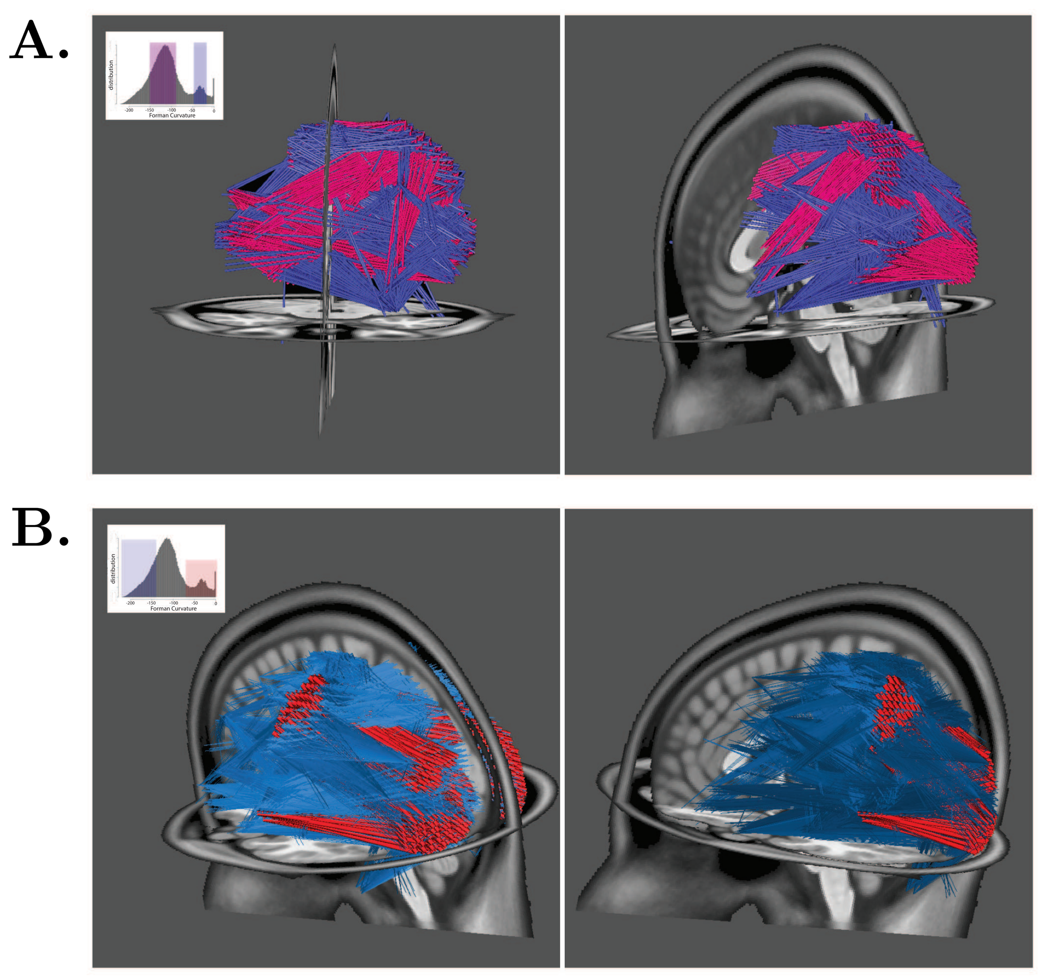


Curvature-colored TED network for motor task (source: Human Connectome project).

- Major network components (communities) are connected by bundles of edges with high curvature. They are displayed in red here; low curvature edges are shown in yellow (A-C).
- The geometry of the network is characterized by these (curvature-wise) dominating edges (**backbone effect**).
- Acting as bridges between major communities they determine the higher order structural organization of the network.

To reduce complexity and make very large networks accessible to computational analysis: Can we reduce the network to these high curvature edges?

3. Neuro-Anatomical Analysis



▲ TED networks with vertices aligned according to the anatomical position of the corresponding voxels.

A: The plot shows two subnetworks consisting of the edges contributing to the two peaks in the distribution (magenta: main peak, blue: secondary peak).

B: Subnetworks consisting of edges with high (blue) and low (red) curvature.

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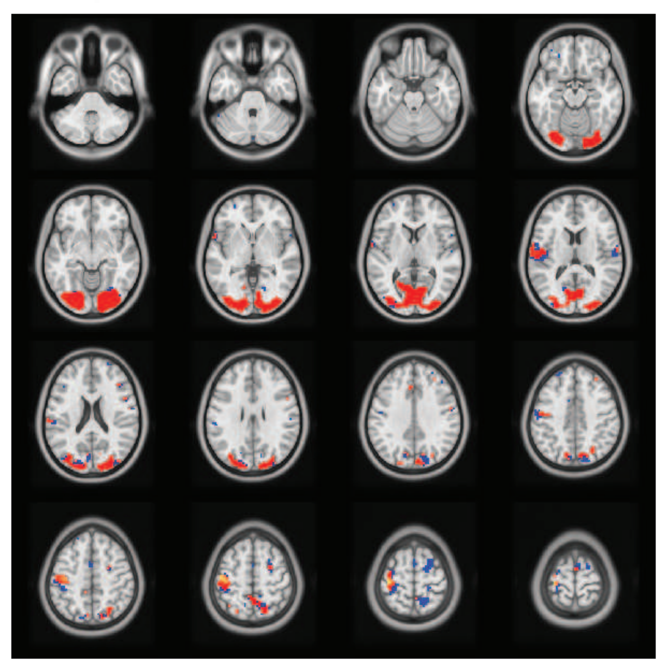
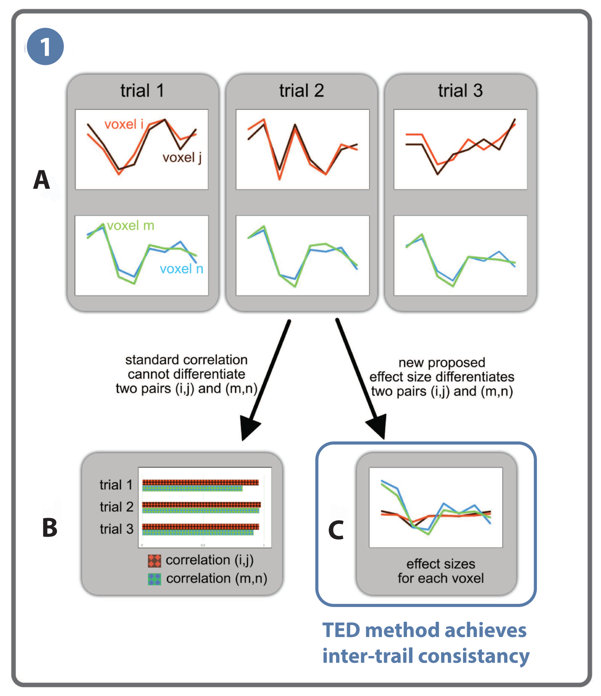
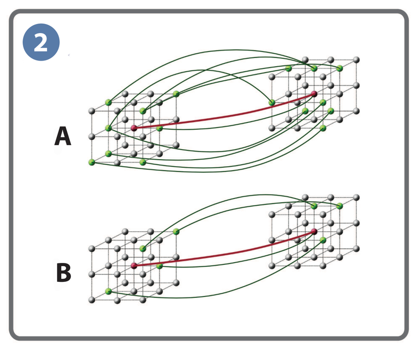
- We analyze task-based fMRI data provided by the Human Connectome Project with a focus on the motor task using minimally preprocessed data (sample size: 50) of the left-right phase-encoding runs.
- The curvature-based analysis of the resulting TED networks reveals subnetworks with activities in distinct regions. Our results suggest that the peaks in the curvature distribution may indicate different functional subnetworks.
- Ongoing work includes a more detailed analysis of these functional subnetworks

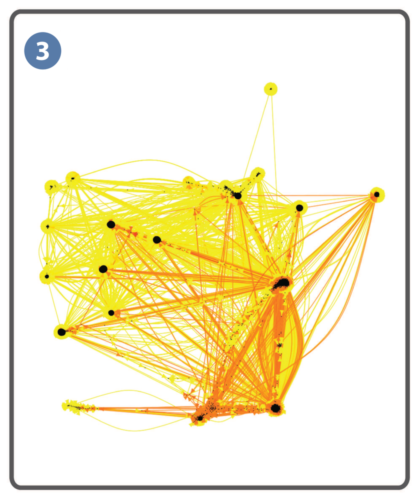


▶ Hubness map for TED subnetworks underlying peak 1 and peak 2. Red areas mark a dominance of peak 1 edges, blue areas a dominance of peak 2 edges.

Methods







1: Comparison of correlation-based approaches (C: TED method).

2: Computation of **t**ask-related **e**dge **d**ensities (**TED**).

3: Curvature-colored brain networks: Nodes are voxels and edges correspond to densities above a given threshold. Edge colors are chosen according to their Forman curvature value.

G. Lohmann et al. Plos One 2016

Future Work

Backbone Effect (ongoing work)

Ricci curvature induces a corresponding geometric flow on the edges, the Ricci flow. Together they characterize the geometry of the network: Edges with high curvature evolve fast under the Ricci flow and determine the higher order network organization (**backbone effect**).

We apply a discrete Ricci flow by iteratively scaling edge weights according to curvature: A reverse Ricci flow acts on the edges and assigns high weights to edges with high curvature and low weights to low curvature edges.

The iterative procedure identifies the backbone of the network and therefore lends itself as a tool for complexity reduction. The much smaller backbone is - in constrast to to the full network - accessble to computational network analysis tools.

Backbone effect: Adaptive weights with reverse Ricci flow

Reverse Ricci flow on edges, induced by Forman-Ricci curvature

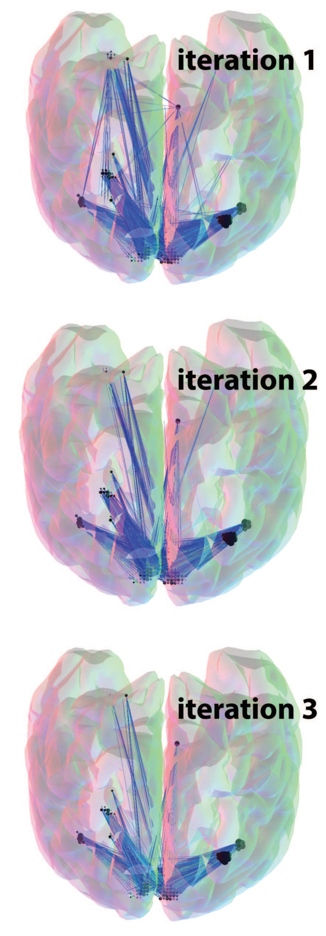
$$\frac{\partial\omega(e,t)}{\partial t}=\mathrm{Ric}_F(\omega(e,t))\omega(e,t)\,;$$

Discretize

$$\omega(e,t+1)=\omega(e,t)+\Delta\cdot\mathrm{Ric}_F(\omega(e,t))\cdot\omega(e,t)\,;$$

Normalize

$$\hat{\omega}(e,t+1)=\frac{\omega(e,t+1)}{\max_{e\in E(G)}\omega(e,t+1)}\,.$$



Test data showing the reduction of the network to high-curvature edges acting as bridges between major network communities.

Conclusions

- We motivate the use of edge-based methods, namely the discrete Forman-Ricci curvature and its associated geometric flow for the analysis of complex brain networks.
- Edges with high Forman curvature span the network acting as bridges between major network communities. This core connectivity structure forms the *backbone* of the network and determins its higher order structural organization. Ongoing work concerns the computation of this backbone through an iterative procedure based on the reverse Ricci flow.
- The neuro-anatomical analysis of the edges underlying peaks in the curvature distribution reveals activities in distinct brain regions. This suggests a correspondance between peaks and different functional subnetworks that we further investigate in ongoing work.

More Info

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