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CORRECTIONS TO: LECTURES ON RESOLUTION OF SINGULARITIES PRINCETON UNIVERSITY PRESS, 2007

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Here is a list of corrections to my book *Lectures on Resolution of Singularities*. Further corrections and comments are most welcome.

Incorrect statements

(1) Proposition 3.9.2 is incorrect. The proof shows that if a resolution that associates X' to X is functorial with respect to étale morphisms then the following holds:

If $Y \to X$ is a smooth morphism then there is a commutative diagram

$$\begin{array}{ccc} Y' & \to & Y \\ \downarrow & & \downarrow \\ X^* & \to & X \end{array}$$

where $Y' \to X^*$ is smooth and $X^* \to X$ is some resolution. (Which may be different from our choice of X'.)

(2) As written, the proof of Theorem 3.36 (Resolution of Singularities III) works only for schemes that are pure dimensional and generically reduced. As far as the existence of resolutions is concerned, this is not a problem. For arbitrary X we can start by normalizing X (or taking the disjoint union of the irreducible components of $\operatorname{red} X$) and then resolving the pure dimensional parts one dimension at a time.

For schemes that are irreducible but not generically reduced, we get a resolution functor but at the last step we need to replace X_n by $\operatorname{red} X_n$; this is not a blow-up.

A similar problem occurs in Theorem 3.35. The blow-up sequence functor $\mathcal{BP}$ commutes with closed embeddings for pure dimensional ambient schemes, but not in general. This is again unlikeley to cause problems in applications.

Mistakes

(1) p.39 two displayed formulas. Two equalities should be inequalities as below:

By Bézout's theorem (cf. [Sha94,IV.2.1]), the degree of $|R|$ is

$$\deg |R| \leq d(d-1) - \sum_i m_i(m_i - 1) = 2g_{\mathrm{app}}(C) + 2(d-1).$$

On the other hand

$$\dim |R| = \dim |A| \geq \binom{d+1}{2} - 1 - \sum_i \binom{m_i}{2} = g_{\mathrm{app}}(C) + 2(d-1).$$

(2) p.44, end of the proof of (1.65.3) is wrong. It should be:

To continue with the induction it is enough to show that

$$\operatorname{mult}_q \Delta' \leq 1 \quad \text{for every } q \in B_p S.$$

This is obvious for $q \notin E$. If $q \in E$ then by (1.40)

$$\operatorname{mult}_q \Delta' \le (E \cdot \Delta') = \operatorname{mult}_p \Delta \le 1$$

and we are done.

(3) p.47, proof of Lemma 1.70 is wrong. Here is a correct proof:

Note that $R\big[\frac{y}{x^a}\big] \subset R + yR\big[\frac{1}{x}\big]$, hence

$$(x^b, y)R\big[\tfrac{y}{x^a}\big] \subset (x^b) + yR\big[\tfrac{1}{x}\big].$$

Thus

$$R \cap (x^b, y)R\big[\tfrac{y}{x^a}\big] \subset R \cap \big((x^b) + yR\big[\tfrac{1}{x}\big]\big) \subset (x^b) + \big(R \cap yR\big[\tfrac{1}{x}\big]\big),$$

where the last inclusion holds since $(x^b) \subset R$. Furthermore, $R \cap yR\big[\frac{1}{x}\big] = (y)$ since x and y are relatively prime, hence

$$(x^b) + \big(R \cap yR\big[\tfrac{1}{x}\big]\big) \subset (x^b) + (y) = (x^b, y).$$

(4) p.74 proof of Theorem 2.12. Insert at the beginning of the proof:

Any cycle Z can be written as $Z^+ - Z^-$ where Z^+, Z^- are both effective and without common irreducible components. Then

$$(Z \cdot Z) = (Z^+ \cdot Z^+) + (Z^- \cdot Z^-) - 2(Z^+ \cdot Z^-) \le (Z^+ \cdot Z^+) + (Z^- \cdot Z^-).$$

Thus it is enough to prove that $(Z \cdot Z) < 0$ for every nonzero effective cycle.

(5) 3.110 on page 193. Comment of Dan Abramovich: The definition of $M(I)$ is not functorial for restriction to open subsets. One needs to allow different components of E^j to come with different coefficients, and then adjust step 3 accordingly.

Clarifications

(1) p.7.line -6: The starting point of the induction, $n = 1$ is the same as the Implicit Function Theorem.

(2) p.22. Remark 1.28: In dimension 1, the converse is proved by (1.30).

(3) p.28 Lemma 1.40: Working over a field k, in part (2), the stronger statement

$$\sum_{p \in C_1 \cap E} \deg\big(k(p)/k\big) \cdot \operatorname{mult}_p C_1 \le \operatorname{mult}_0 C$$

also holds and is more useful.

(4) p.45, Theorem (1.67). The proof also works for imperfect residue fields, if we use the above stronger form of (1.40.2). Indeed, after one blow-up the multiplicity drops unless we have only 1 infinitely near point and the residue field remains unchanged. The rest of the proof works as before.

Typos

(1) p.5.line -4: $y = \epsilon x^{1/m} \cdots$ should be $y = \epsilon^i x^{1/m} \cdots$

(2) p.46 line 5 of (1.69). $y_{k+1} = y_k - a_k x^k$ should be $y_{k+1} = y_k - a_{k+1} x^{k+1}$.

(3) p. 155 line -4: S should be I

(4) p. 180, middle displayed formula, $(\lambda_i b)^s$ should be $(\lambda_i b)^s \cdot \frac{1}{s!}$.

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