MATH 202B, MIDTERM EXAM
Time: 90 minutes

Your name (print): . . . .

Please show all work. Books, notes and calculators are not permitted on this exam.

Write below and **sign** the Pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

1. *(12 points)* Consider the system

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 + 2x_2 + x_3 &= 3 \\ x_1 + x_2 + (a^2 - 5)x_3 &= a \end{aligned}$$

For what values of a does this system have solutions? For what values of a does it have exactly one solution? For what values of a does it have infinitely many solutions?

2. *(18 points)* Find a matrix A for the following linear transformations:
(a) In $\mathbf{R}^2$, reflection across the line $3x = 4y$ followed by rotation through 90 degrees counterclockwise.

(b) In $\mathbf{R}^3$, orthogonal projection onto the plane $x + y + z = 0$.

(c) In $\mathbf{R}^2$, the transformation that sends

$$\begin{bmatrix} -1 \\ 5 \end{bmatrix} \mapsto \begin{bmatrix} 1 \\ 8 \end{bmatrix} \text{ and } \begin{bmatrix} 5 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

3. *(12points)* Find a basis for the row space of A, the column space of A and the kernel of A if

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 & 1 & 2 \\ -1 & -2 & 1 & 2 & 3 & 6 \\ 0 & 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{bmatrix}$$

4. *(18 points)* Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

(a) Find an orthonormal basis for $V = image(A)$ and find the QR factorization of A.

(d) Find a basis for the orthogonal complement of V.

5. *(12 points)* Given any three linearly independent vectors $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$, define

$$\vec{w}_1 = \vec{v}_1 + \vec{v}_2, \quad \vec{w}_2 = \vec{v}_2 + \vec{v}_3, \quad \vec{w}_3 = \vec{v}_1 + \vec{v}_3$$

Show that the vectors $\vec{w}_1$, $\vec{w}_2$ and $\vec{w}_3$ are also linearly independent.