MATH 202B, Quiz 2
Time: 60 minutes

1. *(12 points)* Find the least squares best fit quadratic function $f(x) = ax^2 + bx + c$ to match the given 4 data points:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

2. *(12 points)* Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and } \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

(a) Find the area of the parallelogram in $\mathbf{R}^4$ that has $\vec{v}_1$ and $\vec{v}_2$ as its edges.
(b) Let V be the plane in $\mathbf{R}^4$ that is spanned by $\vec{v}_1$ and $\vec{v}_2$. Find a basis for the orthogonal complement of V.

3. *(12points)*
(a) Use elementary row operations to find

$$\det \begin{bmatrix} 0 & 1 & 3 & -3 \\ 0 & 0 & 4 & -2 \\ -2 & 0 & 4 & -7 \\ 4 & -4 & 4 & 15 \end{bmatrix}$$

(b) A square $n \times n$ matrix is called skew-symmetric if $A^T + A = 0$. Prove that if A is skew-symmetric and if n is odd, then A is not invertible.

4. *(12points)* Find all eigenvalues and compute the corresponding eigenspaces for

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & -5 \\ 4 & 5 & -8 \end{bmatrix}$$

Determine the algebraic and geometric multiplicity for each eigenvalue. Does A have an eigenbasis?

5. *(12points)* Suppose that A is a 2×2 matrix with eigenvalues $\lambda_1 = 0.5$ and $\lambda_2 = 0.2$ with corresponding eigenvectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

(a) Find a solution in closed form to the dynamical system $\mathbf{x}(t+1) = A\mathbf{x}(t)$ given that $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.
(b) Sketch the phaseportrait for this dynamical system.