MATH 202B, Quiz 2
Time: 60 minutes
Due: Friday(December 1) at 3pm the latest in Rm 403 Fine Hall

Your name (print): .................................................................

Please show all work. Books, notes and calculators are not permitted on this exam. Do not discuss the quiz with anyone until after the 3PM Friday deadline.

Write below and sign the Pledge: I pledge my honor that I have not violated the Honor Code during this examination.
1. (12 points) Compute

\[ \det \begin{bmatrix} 1 & 3 & 0 & 2 \\ 2 & 3 & -2 & 9 \\ 3 & 3 & -3 & 10 \\ -1 & 0 & 1 & 0 \end{bmatrix} \]

2. (12 points) Find the area of the parallelogram in \( \mathbb{R}^4 \) that has vertices

\[
\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix},
\]
3. (12 points)
a. Find all eigenvalues and eigenvectors of the matrix

\[
A = \begin{bmatrix}
1 & -2 \\
2 & 1 \\
\end{bmatrix}
\]

b. Solve the dynamical system \( \mathbf{x}(t + 1) = A\mathbf{x}(t) \) and sketch the phaseportrait for this system.
4. (15 points) Does $\mathbb{R}^3$ have a basis consisting entirely of eigenvectors of $A$ if

(a) $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 0 & 3 & 0 \end{bmatrix}$
5. (12 points) Let $A$ be a $2 \times 2$ matrix with eigenvalues $\lambda_1 = 0.5$ and $\lambda_2 = 1.2$ and corresponding eigenvectors $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

a. Solve the dynamical system $\mathbf{x}(t + 1) = A\mathbf{x}(t)$ if $\mathbf{x}(0) = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$.

b. Sketch the phaseportrait for this dynamical system.

c. What is the image of the unit disk under the transformation $T(\mathbf{x}) = A\mathbf{x}$? What is the area of the transformed disk?
6. (16 points) (Justify your answers!)

a. (True or False) If $A$ and $B$ are $n \times n$ matrices then $\det A + \det B = \det (A + B)$.

b. If $A$ is a $3 \times 3$ matrix and $A^3 = I$, then what can you say about the eigenvalues of $A$?

c. (True or False) $\det(-A) = \det A$ for any square matrix $A$.

d. What are the eigenvalues of the matrix of projection onto the plane $x + 2y + 3z = 0$ in $\mathbb{R}^3$?