MATH 202B, Quiz 2
Time: 60 minutes
Due: Friday(December 1) at 3pm the latest in Rm 403 Fine Hall

Your name (print): . . . . . .

Please show all work. Books, notes and calculators are not permitted on this exam. Do not discuss the quiz with anyone until after the 3PM Friday deadline.

Write below and **sign** the Pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

1. *(12 points)* Compute

$$\det \begin{bmatrix} 1 & 3 & 0 & 2 \\ 2 & 3 & -2 & 9 \\ 3 & 3 & -3 & 10 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

2. *(12 points)* Find the area of the parallelogram in $\mathbf{R}^4$ that has vertices

$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix},$$

3. *(12 points)*
a. Find all eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

b. Solve the dynamical system $\mathbf{x}(t+1) = A\mathbf{x}(t)$ and sketch the phaseportrait for this system.

4. *(15 points)* Does $\mathbf{R}^3$ have a basis consisting entirely of eigenvectors of A if

$$(a) \quad A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad (b) \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 0 & 3 & 0 \end{bmatrix}$$

5. *(12 points)* Let A be a 2×2 matrix with eigenvalues $\lambda_1 = 0.5$ and $\lambda_2 = 1.2$ and corresponding eigenvectors $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

a. Solve the dynamical system $\mathbf{x}(t+1) = A\mathbf{x}(t)$ if $\mathbf{x}(0) = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$.

b. Sketch the phaseportrait for this dynamical system.

c. What is the image of the unit disk under the transformation $T(\mathbf{x}) = A\mathbf{x}$? What is the area of the transformed disk?

6. *(16 points)* (Justify your answers!)
a. (True or False) If A and B are $n \times n$ matrices then $\det A + \det B = \det(A + B)$.

b. If A is a 3×3 matrix and $A^3 = I$, then what can you say about the eigenvalues of A?

c. (True or False) $\det(-A) = \det A$ for any square matrix A.

d. What are the eigenvalues of the matrix of projection onto the plane $x + 2y + 3z = 0$ in $\mathbf{R}^3$?