1. Find all vectors $\vec{b}$ so that the system

$$\begin{aligned} x_1 + 2x_2 - x_3 &= b_1 \\ 2x_1 - x_2 + 3x_3 &= b_2 \\ 3x_1 + x_2 + 2x_3 &= b_3 \end{aligned}$$

is solvable. Interpret your answer geometrically. For those choices of $\vec{b}$ that make the system solvable, interpret the solution geometrically.

2. If the matrix A can be transformed to the matrix B by the sequence of row operations given below:

- add twice row 1 to row 2
- divide row 3 by 5
- switch row 2 and row 3

then can B be transformed to A by a sequence of row operations? Explain.

3. (a) If A is a 4×5 matrix of rank 3, then what can you say about solutions to $A\vec{x} = \vec{b}$? about $A\vec{x} = \vec{0}$?

(b) If A is a 7×5 matrix of rank 5, then what can you say about solutions of the system $A\vec{x} = \vec{b}$?

4. The vectors $\vec{x}, \vec{y}, \vec{u}$ and $\vec{v}$ are pictured below. Use the diagram to write $\vec{x}$ and $\vec{y}$ as a linear combination of $\vec{u}$ and $\vec{v}$, or explain why this is impossible.

5. (a) If $P(\vec{x})$ is the orthogonal projection of $\vec{x}$ onto the line $2x_1 = 3x_2$, then find the matrix for P.

(b) Find the matrix for the transformation that reflects $\vec{x}$ across the line $2x_1 = 3x_2$.

(c) Show that the matrix $\begin{bmatrix} 3/5 & 4/5 \\ -1/5 & 7/5 \end{bmatrix}$ defines a shearing transformation.

(d) Which of the transformations above are invertible? For those that are, find the inverse transformations.