MATH 202B, MIDTERM EXAM
Time: 90 minutes

Your name (print): . . . .

Please show all work. Books, notes and calculators are not permitted on this exam.

Write below and **sign** the Pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

1. *(15 points)* Consider the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ where

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 2 & 2 & 4 & 2 \\ -1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(a) T is a linear transformation from ____-space to ____-space.
(b) Find a basis for the kernel of T.
(c) Find a basis for the image of T.
(d) What is the dimension of the row space of A?

2. *(15 points)* **(a)** Describe the geometric effect of the linear transformation

$$T(\mathbf{x}) = \begin{bmatrix} 5 & -12 \\ 12 & 5 \end{bmatrix} \mathbf{x}$$

(b) Find the matrix of orthogonal projection onto the plane $2x - y + 2z = 0$.

3. *(16 points)* Determine whether the following statements are true or false. Justify your answer.

(a) If the reduced row echelon form of A has a zero row then the linear system $A\mathbf{x} = \mathbf{b}$ will have no solutions.

(b) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors in n-space, then so is the set $\{\mathbf{v}_1 + 2\mathbf{v}_2, \mathbf{v}_3 - \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3\}$.

(c) If a 2×2 matrix A is its own inverse, then either $A = I$ or $A = -I$.

(d) The matrix $\begin{bmatrix} -11 & 9 \\ -16 & 13 \end{bmatrix}$ defines a shear.

4. *(14 points)* Use Gram-Schmidt to find an orthonormal basis for the image of A where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

5. *(15 points)* Let V be the subspace of 4-space spanned by

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix} \text{ and } \mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

Find a basis for the orthogonal complement of V.