

**MATH 202B, FINAL EXAM**  
**Time: 3 hours**

**Your name** (print): . . . . .

*Please show all work. Books, notes and calculators are not permitted on this exam.*

**This exam consists of 11 questions, each of which is worth 15 points. You may CHOOSE 10 of these 11 questions. Please indicate clearly which question you choose to omit.**

**Write** below and **sign** the Pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

1. (15points) Find the traffic flow in the system of one-way streets shown below. Is the solution unique?

**2.** (15points) (a) Find the matrix for projection onto the plane  $x - y + z = 0$  relative to the basis  $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

(b) What is the matrix of this transformation in standard coordinates?

**3.** (15points) Find a basis for the image of  $A$ , the kernel of  $A$  and for the orthogonal complement of the kernel of  $A$  if

$$A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 2 & 1 & 1 & 7 \\ 1 & 1 & -1 & 3 \\ 0 & 2 & -6 & -2 \end{bmatrix}$$

4. (15points) Let  $V$  be the vector space in  $\mathbf{R}^4$  spanned by

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ 2 \\ -5 \\ 0 \end{bmatrix}, \text{ and } \mathbf{v}_3 = \begin{bmatrix} 3 \\ 0 \\ 3 \\ -3 \end{bmatrix}$$

(a) Find an orthonormal basis for  $V$ .

(b) Is the vector  $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$  in  $V$ ? If yes, then find its coordinates with respect to your orthonormal basis from (a). In no, then find the vector in  $V$  that is closest to  $\mathbf{b}$ .

5. (15points) (a) Graph the curve  $3x^2 - 4xy + 6y^2 = 20$ .

(b) True or False (Justify your answer): If  $A$  is symmetric and  $\det A > 0$  then  $A$  is positive definite.

**6.** (15points) Find the singular value decomposition of

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

What is the image of the unit disk  $x^2 + y^2 \leq 1$  under the map  $T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = A \begin{bmatrix} x \\ y \end{bmatrix}$ ?

7. (15points) (a) Sketch the phase portrait for the discrete dynamical system

$$\mathbf{x}(t+1) = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix} \mathbf{x}(t)$$

(b) Solve the continuous dynamical system below given that  $x_1(0) = 5$  and  $x_2(0) = 3$ .

$$dx_1/dt = 4x_1 - 2x_2$$

$$dx_2/dt = x_1 + x_2$$



**8.** *(15points)* Find the line of best fit to the data points  $(-1, 4)$ ,  $(0, 3)$ ,  $(1, 1)$  and  $(2, 0)$ .

**9. (15 points)** What is the matrix of the linear transformation  $T : \mathbf{R}^{2 \times 2} \rightarrow \mathbf{R}^{2 \times 2}$  defined by

$$T(A) = A \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$$

relative to the standard basis on  $\mathbf{R}^{2 \times 2}$ ? Find a basis for the kernel of  $T$ .

**10.** *(15points)* Prove that similar matrices have the same eigenvalues and that the corresponding eigenspaces have the same dimension.

**11.** (15points) True or False? Justify your answers.

(a) If  $\text{rref}(A)$  has a zero row, then the system  $A\mathbf{x} = \mathbf{b}$  will have infinitely many solutions for some choice of  $\mathbf{b}$ .

(b) If  $A^2 - A - 6I = 0$  then either  $\lambda = 3$  or  $\lambda = -2$  is an eigenvalue of  $A$ .

(c) The rank of  $A^T A$  is equal to the rank of  $A$ .