HURWITZ NUMBERS AND MONODROMY REPRESENTATIONS

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Abstract

Given Riemann surfaces $X$ and $Y$ where $Y$ is compact, suppose we want to count the number of maps $X \to Y$ which are ramified over some given points and not any others. The answer to such a question is called a Hurwitz number. Given a degree $d$ cover of Riemann surfaces $f : X \to Y$ with branch locus $B$, there is a group homomorphism that sends loops in $Y \setminus B$ to the symmetric group $S_d$ of $d$ elements. This homomorphism is called a monodromy representation, and another problem of interest is to count the number of certain monodromy representations. In this expository talk, I introduce Hurwitz numbers and monodromy representations and explain a neat relation between these two counting questions, linking together multiple areas of mathematics. I will define the terms needed and have tried to make the prerequisites minimal so that this talk should be accessible to a wide audience.