Worksheet 9-6

Exercise 1 (1.8 # 24) An *affine transformation* $T : \mathbb{R}^n \to \mathbb{R}^n$ has the form $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$ with A and $m \times n$ matrix and $\mathbf{b} \in \mathbb{R}^m$. Show that T is *not* a linear transformation when $\mathbf{b} \neq 0$.

Exericise 2 (1.8 # 28) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^n$. It can be shown that the set P of all points in the parallelogram determined by $\mathbf{u}$ and $\mathbf{v}$ has the form $a\mathbf{u}+b\mathbf{v}$ for $0 \leq a \leq 1$ and $0 \leq b \leq 1$. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Explain why the image of a point in P under the transformation T lies in the parallelogram determined by $T(\mathbf{u})$ and $T(\mathbf{v})$.

Exercise 3 (1.9 # 29-30) Describe the possible echelon forms of the standard matrix for a linear transformation T in the following situations: (a) $T : \mathbb{R}^3 \to \mathbb{R}^4$ is one-to-one and (b) $T : \mathbb{R}^4 \to \mathbb{R}^3$ is onto.

Exericise 4 (1.9 # 35) If a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ maps $\mathbb{R}^n$ *onto* $\mathbb{R}^m$, can you give a relation between m and n? If T is one-to-one, what can you say about m and n?

Exercise 5 (1.9 # 36) Why is the question “Is the linear transformation T onto?” an existence question?

Exericise 6 (2.1 # 7-8) If a matrix A is 5×3 and the product AB is 5×7, what is the size of B? How many rows does C have if CD is a 5×4 matrix?

Exercise 7 (2.1 # 24) Suppose A is a $3 \times n$ matrix whose columns span $\mathbb{R}^3$. Explain how to construct an $n \times 3$ matrix D such that $AD = I_3$.

Exericise 8 (2.1 # 25) Suppose A is an $m \times n$ matrix and there exist $n \times m$ matrices C and D such that $CA = I_n$ and $AD = I_m$. Show that $m = n$ and $C = D$.