Worksheet 9-13

Exercise 1 (4.3 # 4, 6, 8) Determine whether the following sets are bases for $\mathbb{R}^3$. For sets that are not bases, determine which are linearly independent and which span $\mathbb{R}^3$.

(a) $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 4 \end{bmatrix}$

(a) $\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \\ 6 \end{bmatrix}$

(a) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$

Exercise 2 (4.3 # 12) Find a basis for the set of vectors in $\mathbb{R}^2$ on the line $y = -3x$.

Exercise 3 (4.3 # 22) True or False?

(a) A linearly indpenedent set in a subspace H is a basis for H.

(b) If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.

(c) A basis is a linearly independent set that is as large as possible.

(d) If B is an echelon form matrix A, then the pivot columns of B form a bssi for Col$A$.

Exercise 4 (4.3 # 26) In the vector space of all real-valued functions, find a basis for the subspace spanned by $\{\sin(t), \sin(2t), \sin(t)\cos(t)\}$.

Exercise 5 (4.4 # 3) Find the vector $\mathbf{x}$ determined by the given coordinate vector $[\mathbf{x}]_B$ and the given basis B.

$$B = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix} \right\} \quad [\mathbf{x}]_B = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

Exercise 6 (4.4 # 10) Find the change-of-coordinates matrix from B to the standard basis of $\mathbb{R}^n$.

$$B = \left\{ \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}$$

Exercise 7 (4.4 # 23/24) Let V be a vector-space, $B = \{b_1, \ldots, b_n\}$ be a basis and $\mathbf{x} \mapsto [\mathbf{x}]_B$ be the coordinate mapping. Show that the coordinate mapping is one-to-one and onto.

Exercise 8 (4.4 # 28,29,30) Use coordinate vectors to test the linear independence of the sets of polynomials. Explain your work.

(a) $1 - 2t^2 - t^3, t + 2t^3, 1 + t - 2t^2$

(b) $(1-t)^2, t - 2t^2 + t^3, (1-t)^3$

(c) $(2-t)^3, (3-t)^2, 1 + 6t - 5t^2 + t^3$