Worksheet 9-13

Exericise 1 (3.2 # 33) Let A and B be square matrices. Show that $\det(AB) = \det(BA)$.

Exercise 2 (3.2 # 35) Let U be a square matrix where $U^T U = I$. Show that $\det(U) = \pm 1$.

Exercise 3 (3.2 # 36) Suppose that A is a square matrix such that $\det(A^4) = 0$. Explain why A can't be invertible.

Exercise 4 (3.3 # 21) Find the area of the parallelogram with vertices $(-1, 0)(0, 5)$, $(1, -4)$, $(2, 1)$.

Exercise 5 (Chapter 3 Supplements # 1a-e) True or False.

(a) If A is a 2×2 matrix with 0 determinant, then one column of A is a multiple of the other.

(b) If two rows of a 3×3 matrix A are the same, then $\det(A) = 0$.

(c) If A is a 3×3 matrix, then $\det(5A) = 5\det(A)$.

(d) If A and B are $n \times n$ matrices with $\det(A) = 2$ and $\det(B) = 3$, then $\det(A + B) = 5$.

(e) If A is $n \times n$ and $\det(A) = 2$ then $\det(A^3) = 6$.

Exercise 6 (Chapter 3 Supplements, # 15) Let A, B, C and D be $n \times n$ and I be $n \times n$ matrices (with I the identity). Use the definition or properties of the determinant to justify the following formulas.

(a) $\det \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} = \det(A)$

(b) $\det \begin{bmatrix} I & 0 \\ C & D \end{bmatrix} = \det(D)$

(c) $\det \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} = \det(A)\det(D) = \det \begin{bmatrix} A & B \\ 0 & D \end{bmatrix}$

Exercise 7 Does there exist a pair of matrices A and B where $AB - BA = I$?