

## Worksheet 9-11

**Exercise 1** (2.1 # 25) Suppose  $A$  is an  $m \times n$  matrix and there exists  $n \times m$  matrices  $C$  and  $D$  such that  $CA = I_n$  and  $AD = I_m$ . Prove that  $m = n$  and  $C = D$ . (Hint: Consider the product  $CAD$ ).

**Exercise 2** Suppose that  $AD = I_m$  (where  $A$  is  $m \times n$ ,  $D$  is  $n \times m$  and  $I_m$  is the  $m \times m$  identity matrix). Show that the linear map  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  given by  $T(x) = Ax$  is onto.

**Exercise 3 (2.3 # 36)** Suppose a linear transformation  $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$  has the property that  $T(\mathbf{u}) = T(\mathbf{v})$  for some pair of distinct vectors  $\mathbf{u}$  and  $\mathbf{v}$ . Can  $T$  map  $\mathbb{R}^n$  onto  $\mathbb{R}^n$ ? Why or why not?

**Exercise 4 (2.3 # 39)** Let  $T$  be a linear transformation that maps  $\mathbb{R}^n$  onto  $\mathbb{R}^n$ . Show that  $T^{-1}$  exists and maps  $\mathbb{R}^n$  to  $\mathbb{R}^n$ . Do not refer to the matrix  $A$  of  $T$  in your argument! Is  $T^{-1}$  also one to one?

**Exercise 5 (3.1 # 10)** Compute the determinant by cofactor expansion. At each step, choose the row or column that minimizes computation.

$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -2 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix}$$

**Exercise 6 (3.1 # 37, sort of)** If  $c \in \mathbb{R}$  and  $c \neq 0$ , is true that  $\det(cA) = c\det(A)$ ? If so, prove it. If not, provide the correct formula and explain why it is correct.

**Exercise 7 (3.2 # 16,17,18)** Find the determinants of  $B, C$  and  $D$  using the determinant of  $A$  when  $A$  is defined as:

$$A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$$

and  $B, C$  and  $D$  are given by:

$$B = \begin{vmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{vmatrix} \quad C = \begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix} \quad D = \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix}$$