Worksheet 10-4

Exercise 1 (5.3 # 8,12,14) Diagonalize the following matrices:

$$\begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & -2 \\ 1 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

Exercise 2 (5.3 # 25) A is a 4×4 matrix with trhee eigenvalues. One eigenspace is one-dimensional and one of the other eigenspaces is two-dimensional. Is it possible that A is not diagonalizable? Prove that it is diagonalizable or give an example where it isn't.

Exercise 3 (5.3 # 27) Show that if A is diagonalizable and invertible, then so is A^{-1}.

Exercise 4 Show that any diagonalizable matrix such that $A^k = 0$ for some $k > 0$ is the 0 matrix. Use this to find an example of a non-diagonalizable matrix.

Exercise 5 (5 Supplements, # 5) If $p(x) = c_0 + c_1 x + \cdots + c_n x^n$, define $p(A)$ to be the matrix formed by replacing each power of x in $p(x)$ with the corresponding power of A (with $A^0 = I$). Show that if λ is an eigenvalue of A, then one eigenvalue of $p(A)$ is $p(\lambda)$.

Exercise 6 Show that if $A = A^T$ then any eigenvectors u and v such that $Au = \lambda u$, $Av = \mu v$ and $\lambda \neq \mu$ will be perpendicular: i.e. $u \cdot v = u^T v = 0$.