## Worksheet 10-25

Exercise 0 Make sure you know what the following terms mean. Give two examples of a basis on $\mathbb{R}^{3}$ and write down what each of these things is in those bases. Please check your understanding with a neighbor!

- basis
- coordinate vector of $v$ in the basis $B$
- coordinate mapping
- change of basis matrix

Exericise 1 Let $B=\left\{b_{1}, \ldots, b_{n}\right\}$ be a basis for $\mathbb{R}^{n}$. Consider the linear transformation $T: \mathbb{R}^{n} \rightarrow \mathbb{R}^{n}$ given by $x \mapsto[x]_{B}$, which takes $x$ to its coordinate vector in the basis $B$. Find the matrix of this transformation with respect to the standard basis on both sides on $\mathbb{R}^{n}$.

Exercise 2 Let $V$ be a vector-space and $B=\left\{b_{1}, \ldots, b_{n}\right\}$ be a basis. Show that the coordinate mapping $T_{B}: V \rightarrow \mathbb{R}^{n}$ is one-to-one and onto.

Exercise 3 Find the eigenvalues and eigenvectors of the following matrices. Write down an eigenbasis for both. Find the change of basis matrix from the eigenbasis of $A$ to the eigenbasis of $B$. Use this to write down the matrix for $A$ in the eigenbasis of $B$.

$$
\left[\begin{array}{ccc}
3 & 1 & 1 \\
0 & 5 & 0 \\
-2 & 0 & 7
\end{array}\right] \quad\left[\begin{array}{ccc}
4 & 0 & -1 \\
0 & 4 & -1 \\
1 & 0 & 2
\end{array}\right]
$$

Exercise 4 Do every true-false question on p. 390 (Ch. 6 Supplementary Exercises).

