Worksheet 10-23

Exericise 1 (7.2 # 6) Find the matrix of the quadratic form. Assume x is in $\mathbb{R}^3$.

$$5x_1^2 + 7x_2^2 - 6x_1x_2 + 4x_1x_3 - 2x_2x_3$$

$$4x_1x_2 + 6x_1x_3 - 8x_2x_3$$

Exercise 2 (7.2 # 12) Classify the quadratic form. Then make a change of variables,$x = Py$, that transforms the quadratic form into one with no cross product term. Write the new quadratic form. Construct P using the methods of Section 7.1.

$$-5x_1^2 + 4x_1x_2 - 2x_2^2$$

Exercise 3 (7.2 # 26) Show that if an $n \times n$ matrix A is positive definite, then there exists a positive definite matrix B such that $A = B^T B$. There is a hint in the book but try not to look.

Exercise 4 (7.2 # 28) Let A be an $n \times n$ symmetric invertible matrix. Show that if the quadratic form $x^T Ax$ is positive definite, then so is the quadratic form $x^T A^{-1} x$.

Exercise 5 Let $B : V \times V \to \mathbb{R}$ be a bilinear form. Given a subspace $U \subset V$, we denote by U^B the "B-perpendicular" to U, i.e.

$$U^B = \{v \in V | B(u, v) = 0\}$$

Find a bilinear form B such that there exists a vector v with $v \in \text{span}(v)^B$. Show that there exists such a v if and only if B is neither positive definite nor negative definite.