Worksheet 10-2

Exercise 1 (5.2 # 4,7) Find the characteristic polynomial and the real eigenvalues of the matrices:

$$\begin{bmatrix} 8 & 2 \\ 3 & 3 \end{bmatrix} \qquad \begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix}$$

Exercise 2 (5.2 # 24) Show that if A and B are similar, then $\det(A) = \det(B)$.

Exercise 3 (5.2 # 27) Let:

$$A = \begin{bmatrix} .5 & .2 & .3 \\ .3 & .8 & .3 \\ .2 & 0 & .4 \end{bmatrix} \quad v_1 = \begin{bmatrix} .3 \\ .6 \\ .1 \end{bmatrix} v_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(a) Show that v_1, v_2, v_3 are eigenvectors of A.

(b) Let x_0 be any vector in $\mathbb{R}^3$ with non-negative entries whose sum is 1. Explain why thre are constants c_1, c_2, c_3 such that $x_0 = c_1v_1 + c_2v_2 + c_3v_3$. Compute $w^T x_0$ and deduce that $c_1 = 1$.

(c) For $k = 1, 2, \ldots$ define $x_k = A^k x_0$ with x_0 as in (b). Show that $x_k \to v_1$ as k increases.

Exercise 4 (Ch. 5 Supplements, # 10) A matrix is diagonaliable if it is similar to a diagonal matrix. Show that if A is diagonalizable with all eigenvalues less than 1 in magnitude, then A^k tends to the 0 matrix as $k \to \infty$.

Exercise 5 True or False.

(a) If A and B are invertible $n \times n$ matrices, then AB and BA are similar.

(b) An $n \times n$ matrix with n linearly independent eigenvectors is invertible.

(c) Two eigenvectors corresponding to the same eigenvalues are always linearly dependent.

(d) Each eigenvector of an invertible matrix A is also an eigenvector of A^{-1}.

Exercise 6 Is every degree n polynomial $p(\lambda)$ in a variable λ the characteristic polynomial of some matrix? Prove or find a counter-example.

Exercise 7 Find an example of a matrix that isn't diagonalizable and prove that it is not diagonalizable.