Worksheet 10-16

Exericise 1 (6.5 # 6) Describe all least squares solutions of the equation Ax = b with:

	1	1	0		7
A =	1	1	0	b =	2
	1	1	0		3
	1	0	1		6
	1	0	1		5
	1	0	1		4

Exercise 2 (6.5 # 19, On The HW) Let A be an $m \times n$ matrix. Show that a vector x in \mathbb{R}^n satisfies Ax = 0 if and only if $A^T A x = 0$, i.e. that $\operatorname{Nul}(A) = \operatorname{Nul}(A^T A)$.

Exercise 3 (6.5 # 21) Let A be an $m \times n$ matrix whose columns are linearly independent.

- (a) Use Exercise 2 to show that $A^T A$ is an invertible matrix.
- (b) What does this say about the shape of A?
- (c) Determine the rank of A.

Exercise 4 (6.5 # 24) Find a formula for the least-square solution of Ax = b when the columns of A are orthonormal.

Exercise 5 Suppose the columns of A are linearly independent, let b be a vector and $c \neq 0$ be a scalar. Find an expression for the least-square solutions to Ax = cb in terms of the least-square solutions of Ax = b.