## Worksheet 10-16

Exericise $1(6.5 \# 6)$ Describe all least squares solutions of the equation $A x=b$ with:

$$
A=\left[\begin{array}{lll}
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 0 & 1 \\
1 & 0 & 1 \\
1 & 0 & 1
\end{array}\right] \quad b=\left[\begin{array}{l}
7 \\
2 \\
3 \\
6 \\
5 \\
4
\end{array}\right]
$$

Exercise 2 (6.5 \# 19, On The HW) Let $A$ be an $m \times n$ matrix. Show that a vector $x$ in $\mathbb{R}^{n}$ satisfies $A x=0$ if and only if $A^{T} A x=0$, i.e. that $\operatorname{Nul}(A)=$ $\operatorname{Nul}\left(A^{T} A\right)$.

Exercise 3 (6.5 \# 21) Let $A$ be an $m \times n$ matrix whose columns are linearly independent.
(a) Use Exercise 2 to show that $A^{T} A$ is an invertible matrix.
(b) What does this say about the shape of $A$ ?
(c) Determine the rank of $A$.

Exercise $4(6.5 \# 24)$ Find a formula for the least-square solution of $A x=b$ when the columns of $A$ are orthonormal.

Exercise 5 Suppose the columns of $A$ are linearly independent, let $b$ be a vector and $c \neq 0$ be a scalar. Find an expression for the least-square solutions to $A x=c b$ in terms of the least-square solutions of $A x=b$.

