Worksheet 10-11

Exercise 1 (6.2 # 5) Determine if the following two sets are orthogonal.

$$\begin{bmatrix} 3 \\ -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 7 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ -4 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -3 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 5 \\ -1 \end{bmatrix}$$

Exercise 2 (6.2 # 12) Compute the orthogonal projection of $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ onto the line theough $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and the origin.

Exercise 3 Prove that n non-zero vectors that form an orthogonal set (i.e. that are all pairwise orthohgonal) in $\mathbb{R}^n$ are a basis.

Exercise 4 (6.2 # 33) Given $u \neq 0$ in $\mathbb{R}^n$, let $L = \text{span}(u)$. Show that the map $x \mapsto \text{proj}_L(x)$ (that is, orthogonal projection onto the line L) is a linear transformation.

Exercise Let L be a line in $\mathbb{R}^n$ and $\text{proj}_L : \mathbb{R}^n \to \mathbb{R}^n$ be projection onto $L \subset \mathbb{R}^n$. Let M be the matrix of this linear transformation, i.e $M = uu^T$Show that there is an orthogonal matrix U such that:

$$UMU^T = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

Exercise 6 (Challenge Problem 1) Show that, given a point $x \in \mathbb{R}^n$ and a line L in $\mathbb{R}^n$, the orthogonal projection $\text{proj}_L(x)$ in L is the closest point in L to x, i.e. it is the point y in L such that $|y - x|$ is the smallest.

Exercise 7 (Challenge Problem 2) Let V be a vector space, and let U and W be complimentary subspaces in V. That means that $U + W = V$ and that the dimensions of U and W are complimentary.

(a) Show that every vector $v \in V$ can be written uniquely as a combination $v = u + w$ with $u \in U$ and $w \in W$.

(b) Define projection $\pi_W : V \to W$ to W by the formula $\pi_W(v) = w$ where $v = u + w$, $u \in U$ and $w \in W$. By part (a), this is unique. Given a basis $\{u_1, \ldots, u_m\}$ of U and a basis $\{w_1, \ldots, w_m\}$ of W, describe how to write down a matrix for π_W.