Math 54 Section 4: Quiz 8

Problem 1 (2 pts each) Give a 1-2 sentence explanation about why the following is true.

- (a) Every positive definite matrix is invertible.
- (b) A diagonal matrix with positive diagonal entries is positive definite.
- (c) A symmettric matrix with a positive determinant might not be positive definite!

Problem 2 (4 pts) Let $W \subset \mathbb{R}^n$ be a subspace and consider the linear map $\operatorname{proj}_W : \mathbb{R}^n \to \mathbb{R}^n$. Show that the matrix A of proj_W is symmetric. Show that the quadratic form $q(x) = x \cdot (Ax)$ is positive semi-definite.

Problem 3 (4 pts) What is the SVD $U\Sigma V^T$ of A if A has orthogonal columns a_1, \ldots, a_n of length c_1, \ldots, c_n ?