Math 54 Section 4: Quiz 7

Problem 1 (2 pt each) True Or False? Justify your answer. Each question has a 1 line justification.

(a) If a square matrix has orthogonal columns then it also has orthogonal rows.

(b) If x is orthogonal to both u and v, then it is orthogonal to $u - v$.

(c) If $\{v_1, \ldots, v_k\}$ is an set of orthonormal vectors then so is $\{c_1 v_1, \ldots, c_k v_k\}$ for scalars c_i.

Problem 2 (4 pts) FInd the least squared solution of $Ax = b$.

$$A = \begin{bmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 3 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Problem 3 (4 pts) Let A be an $m \times n$ matrix. Show that $\mathrm{Col}(A)^{\perp} = \mathrm{Null}(A^T)$.

Problem 4 (4 pts) Let V be an inner product space with inner product $\langle \cdot, \cdot \rangle$. Let $\text{proj}_W : V \to V$ be orthogonal projection to a subspace $W \subset V$. What are the eigenvalues of proj_W (2 pts)? Prove it (2 pts).

Problem 5 (4 pts) Let A be a symmetric matrix, i.e. where $A = A^T$. Let u and v be eigenvectors with real eigenvalues λ and μ, such that $\lambda \neq \mu$. Show that u and v are orthogonal.