

Math 54 Section 4: Quiz 5

Problem 1 (1 pt each) True Or False? Justify your answer. Each question has a 1 line justification.

- (a) If A is similar to B , then A^2 is similar to B^2 .
- (b) A square matrix with characteristic polynomial $p(\lambda) = \lambda^n$ is the identity matrix.
- (c) If $\|u - v\|^2 > \|u\|^2 - \|v\|^2$, then u and v are not orthogonal.
- (d) There exists a vector $v \in \mathbb{R}^n$ and a line L such that $\|\text{proj}_L(v)\| > \|v\|$.

Problem 2 (2 pts) Let $T : \mathbb{P}_2 \rightarrow \mathbb{R}^3$ be the map:

$$T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(1) \\ \mathbf{p}(2) \end{bmatrix}$$

Find the matrix for T relative to the basis $\{-1, t+1, t^2+t\}$ on \mathbb{P}_2 and the standard basis for \mathbb{R}^3 .

Problem 3 (2 pts) Determine if the following set is orthogonal.

$$T(\mathbf{p}) = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{20} \\ 3/\sqrt{20} \end{bmatrix} \quad \begin{bmatrix} 3/\sqrt{10} \\ -1/\sqrt{20} \\ -1/\sqrt{20} \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

Problem 4 (2 pts) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be a linear transformation that preserves length. That is, $\|T(v)\| = \|v\|$. Show that the dot product is also preserved, $T(u) \cdot T(v) = u \cdot v$.

Problem 5 (2 pts) Show that if square matrices U and V preserve length, then so does UV . Use this to show that if U preserves length, then any matrix gotten by permuting the rows and columns also preserves length. (Hint: use permutation matrices).