Math 54 Section 4: Quiz 5

Problem 1 True or False?

(a) If A is invertible and 1 is an eigenvalue for A, then 1 is also an eignevalue for A^{-1}.

(b) Each eigenvalue of A is also an eigenvalue for A^2.

(c) Two eigenvectors corresponding to the same eigenvalue are always linearly independent.

(d) Similar matrices always have the exact same eigenvalues

(e) If a 5×5 matrix A has fewer than 5 distinct eigenvalues, then A is not diagonalizable.

(f) There is a 2×2 matrix that has no eigenvectors in $\mathbb{R}^2$.

Problem 2 Find the eigenvalues and eigenvectors of the matrix:

$$A = \begin{bmatrix} 4 & 1 \\ 3 & 6 \end{bmatrix}$$

Problem 3 Show that a non-invertible $n \times n$ matrix has zero as an eigenvalue.

Problem 4 Suppose that $p(t)$ is a polynomial with real coefficients (not complex). Then for every complex root $\lambda \in \mathbb{C}$ such that $p(\lambda) = 0$, $p(\bar{\lambda}) = 0$ as well. Here $\bar{\lambda}$ is the complex conjugate of λ. Use this to show that every real $n \times n$ matrix A has a real eigenvalue when n is odd.

Problem 5 Suppose that A is diagonalizable. Let $p(t) = \det(tI - A) = \sum_{i=0}^{n} a_i t^i$ be the characteristic polynomial, given as a sum. Let $p(A)$ be the matrix gotten by plugging t in for A, i.e.

$$p(A) = \sum_{i=0}^{n} a_i A^i$$

Show that $p(A)$ is the zero matrix. (Warning: $\det(AI - A) = \det(A - A) = 0$ is not a proof!).